RESOLUTION OF TEMPERATURE PROBLEMS BY THE USE OF FINITE FOURIER TRANSFORMATIONS

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1. The finite sine transformation of the convolution. The finite sine transformation and the finite cosine transformation of $U(x)$ with respect to x are defined[2] by

$$u_s(n) = \int_0^\pi U(x) \sin nx dx = S\{U(x)\}$$

and

$$u_c(n) = \int_0^\pi U(x) \cos nx dx = C\{U(x)\},$$

respectively.[3] In particular, $S\{U'\} = -nC\{U\}$, and $S\{U''\} = -n[(-1)^n U(\pi) - U(0)] - n^2 S\{U\}$. Further, if $U(x)$ and $U'(x)$ vanish at the end points of the interval $(0, \pi)$, then $S\{U''\} = -n^2 S\{U\}$, and $C\{U'\} = nS\{U\}$.

The convolution $U_1 * U_2$, or Faltung,[2] of the two functions $U_1(x)$, $-2\pi \leqq x \leqq 2\pi$, and $U_2(x)$, $-\pi \leqq x \leqq \pi$, is defined as follows:

$$U_1 * U_2 = \int_{-\pi}^{\pi} U_1(x - \xi) U_2(\xi) d\xi.$$

It is evident that the convolution of two even functions is an even function, that the convolution of two odd functions is an even function, and that the convolution of an odd function and an even function is an odd function.

The following theorem is proved by Kniess:[4]

THEOREM. *If $U_1(x)$, $-2\pi \leqq x \leqq 2\pi$, and $U_2(x)$, $-\pi \leqq x \leqq \pi$, are bounded and integrable, if U_1 is odd and periodic with period 2π, and if U_2 is even, then*

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[2] G. Doetsch, *Integration von Differentialgleichungen vermittels der endlichen Fourier Transformation*, Math. Ann. vol. 112 (1935) pp. 52–68.

[3] The lower case letters will be used to signify the transforms of the functions designated by the corresponding capital letters. Instead of using the symbols $u_s(n)$ and $u_c(n)$ we shall use $u(n)$ for both, whenever it is evident which one is meant.

[4] Hans Kniess, *Lösung von Randwertproblemen bei Systemen gewöhnlicher Differentialgleichungen vermittels der endlichen Fourier Transformation*, Math. Zeit. vol. 44 (1938) pp. 266–291.