tion of the random variables x and y where x corresponds to the values assumed by the unknown under test and y corresponds to the values assumed by the control from test to test. (Received March 23, 1944.)

163. P. R. Halmos: *Randon alms.*

Suppose that a pound of gold dust is distributed at random among a countably infinite set of beggars, so that the first beggar gets a random portion of the gold, the second beggar gets a random portion of the remainder, and so on ad infinitum. The purpose of this work is to calculate the actual and the asymptotic distributions of x_n and S_n (where x_n is the amount received by the nth beggar and $S_n = \sum_{i \leqq n} x_i$) and also to study the rate of convergence of the random series $x_1+x_2+x_3+ \cdots$, under the assumption that the phrase "random portion," occurring an infinite number of times in the description of the stochastic process, receives the same interpretation each time. The results may be interpreted as properties of a random distribution of a unit mass on the positive integers; they may be used to explain the experimentally observed distribution of sizes of mineral grain particles; and they occur also as distributions of energy in the theory of scattering of neutrons by protons of the same mass. (Received March 24, 1944.)

164. Henry Scheffé and J. W. Tukey: *Contributions to the theory of non-parametric estimation.*

For problems of non-parametric estimation, concerning an unknown cumulative distribution function $F(x)$, three good solutions are available: (i) confidence intervals for the median of F (W. R. Thompson, K. R. Nair), (ii) tolerance limits for F (Wilks), and (iii) confidence limits for F (Wald, Wolfowitz, Kolmogoroff). Heretofore (i) and (ii) have been limited to the case where $F'(x)$ is known to be continuous. By means of a theorem of general application they are extended to the case where F need only be continuous. The only previous result for discontinuous F is that of Kolmogoroff for (iii). The appropriate modifications of (i) and (ii) extending their validity to this case are found. Some uniqueness results limiting the kind of statistics usable in such problems are obtained. Sufficiently complete tables for applying (i) and (iii) have been published, but computations for (ii) have been extremely few and laborious. A simple formula based on the z and χ^2-distributions is found which gives highly accurate approximations in ranges of practical interest. All the results for (ii) also apply to Wald's tolerance intervals for the multivariate case. (Received March 14, 1944.)

Topology

165. E. F. Beckenbach and R. H. Bing: *On generalized convex functions.*

Let $F(x; \alpha, \beta)$ be a two-parameter family of real continuous functions defined in an interval (a, b) such that there is a unique member of the family taking on arbitrary values y_1, y_2 at arbitrary distinct points x_1, x_2 of the interval. A real function $f(x)$ is said to be a sub-$F(x; \alpha, \beta)$ function in (a, b) provided at the midpoint x_0 of each subinterval I of (a, b) we have $f(x_0) \leqq F(x_0; \alpha, \beta)$ for that $F(x; \alpha, \beta)$ which coincides with $f(x)$ at the endpoints of I. The family $F(x; \alpha, \beta)$ is not necessarily topologically equivalent to the set of non-vertical line segments in the strip; hence the study of sub-$F(x; \alpha, \beta)$ functions is not topologically equivalent to the study of convex functions. It is shown among other things that if a sub-$F(x; \alpha, \beta)$ function $f(x)$ is bounded,