characterization is in general equivalent to the following: On the variety there exist r families of curves such that when the tangent r-plane is displaced along a curve of the family its intersection with a neighboring tangent r-plane is the tangent $(r-1)$-plane formed by the tangents of the other $r-1$ curves. Basing on this property one can define on each of these r tangents AA_i, $i=1, \cdots, r$, $r-1$ points A_{ij}, $j \neq i$, having the property that when the s-plane $AA_{i_1} \cdots A_{i_s}$ is displaced along AA_j, $j \neq i_1, \cdots, i_s$, its intersection with a neighboring s-plane is the $(s-1)$-plane $A_{i_1 j} \cdots A_{i_s j}$. The points A_{ij} describe varieties of the same type and are naturally defined as the Laplace transforms of the given variety, there being altogether $r(r-1)$ transforms. Many well known properties of Laplace transforms can be generalized. (Received February 23, 1944.)

160. S. B. Jackson: *Vertices of plane curves.*

A closed curve of class C'', not a circle, has two vertices by the continuity of the curvature. The present paper seeks to characterize geometrically those curves with exactly two vertices. Let a curve be called normalized if it contains no complete circles, and let a simple closed arc of the curve which is never crossed by the curve be called a simple loop. The following facts are established for any normalized curve C with two vertices: (a) C may be divided into two simple arcs; (b) all double points are simple; (c) C contains exactly two simple loops, one containing each vertex; (d) none of the plane regions bounded by C are bounded always in the same sense except those regions bounded by the loops; (e) at any points of tangency the directed tangents coincide. For a curve which is not normalized these results are modified slightly. Two familiar theorems regarding the number of vertices on an oval are generalized to any simple closed curve. The methods employed are entirely elementary, extensive use being made of the invariance of vertices under direct circular transformations. (Received February 4, 1944.)

161. J. E. Wilkins: *The contact of a cubic surface with a ruled surface.*

It is shown that there exist ∞^1 cubic surfaces having contact of order 5 with a nondevelopable ruled surface. If there is any cubic surface having contact of order 6 with a nondevelopable ruled surface, then the surface is itself a cubic surface. In order to obtain these results, there are first derived power series expansions for a nondevelopable ruled surface to terms of the sixth degree. Similar investigations are made for developable surfaces. (Received April 1, 1944.)

Statistics and Probability

162. C. W. Churchman and Benjamin Epstein: *Estimates of error in parallel experiments.*

It is common in many types of tests to have not only a random error from test to test due to a large number of unallocable causes, but it is also possible to have systematic errors present. It is because of this possibility that one tests not only samples of the unknown, but also control samples. The purpose of using control samples is two-fold—(a) to find out whether or not abnormal experimental conditions exist during the test and (b) to establish tentatively a level for the particular test under consideration. It is shown that a statistic can be found which gives the most efficient estimate of the corrections to be applied to the unknown under test for a variety of experimental conditions. It is further shown that this statistic must be a linear func-