156. J. A. Shohat: *Series expansions for the periodic solution of Van der Pol's equation and its frequency for all values of the parameter.*

If the parameter μ in Van der Pol's equation $d^2u/dt^2 - \mu(1-u^2)du/dt + u = 0$ is small, power series expansions for the periodic solution u (unique, save for time-translation) and its frequency v can be and have been given, say, by Lidstedt's method. In the present paper the author gives (for the first time, he believes) series expansion for u and v, valid for all values of μ—large and small. Numerical computation agrees quite well with known numerical results. (Received March 13, 1944.)

GEOMETRY

157. Reinhold Baer: *The fundamental theorems of elementary geometry. An axiomatic analysis.*

It is the object of this paper to evaluate the logical interdependence of certain fundamental theorems in elementary geometry. The paper deals with the theorems asserting the copunctuality of each of the following triplets of lines: medians, altitudes, perpendicular bisectors, and bisectors of angles; and the theorem stating that the locus of the points of equal distance from two different points is a line. The framework of our discussion is provided by a general affine plane in which we introduce just as many further relations as are needed for stating the investigated theorems. (Received March 22, 1944.)

158. P. O. Bell: *A study of surfaces by means of a system of differential equations of the first order.*

The projective differential geometry of a surface in ordinary space is studied by means of tetrads of surfaces whose corresponding points x_i ($i = 0, 1, 2, 3$) are linearly independent. The general homogeneous coordinates of x_i satisfy a system of equations $\partial x_i/\partial u^\alpha = C_{h,i\alpha}x_h$, $\alpha = 1, 2$, summed for $h = 0, 1, 2, 3$. With the points x_i as vertices of a local reference tetrahedron an algebraic surface $a_{ij\ldots l}x^i x^j \cdots x^l = 0$ is fixed as u^1, u^2 vary independently, if and only if the coefficients $a_{ij\ldots l}$ are proportional to the corresponding components of the covariant derivatives, of the aggregate of these coefficients, with respect to the connection $C_{h,i\alpha}$. Such conditions of immovability form the basis for a general theory of envelopes. Tetrads of surfaces are first investigated. The study of a surface S_0 is then undertaken by specializing the general theory. Auxiliary surfaces S_1, S_2, S_3 covariantly determined with respect to S_0 are selected so that the fundamental differential equations are as simple as possible and exhibit desirable properties of symmetry. Some differential invariants are characterized geometrically. When the asymptotic curves are parametric on one of the surfaces one of these invariants becomes the projective linear element and another becomes Fubini's element of projective arc length. (Received April 1, 1944.)

159. S. S. Chern: *Laplace transforms of a class of higher dimensional varieties in a projective space of n dimensions.*

In a projective space of n dimensions a class of r-dimensional varieties is defined, which form a natural generalization of the surfaces sustaining conjugate nets. These varieties are characterized by the property that the asymptotic net is an $(r-1)$-parameter linear system of cones whose base cones are linear spaces counted twice. (See E. Cartan, Bull. Soc. Math. France vol. 47 (1919) pp. 125–160.) This geometrical