

of treating difference equations is established and can be applied when  $b_{ij}(x) \equiv 0$ , and also when  $p(x)$  is replaced by 0 in (1). The method makes use of analytic implicit functions and a matrix transformation, and the case where  $b_{ij}(x) \neq 0$  is made to depend upon the case  $b_{ij}(x) \equiv 0$ . (Received February 1, 1944.)

149. Otto Szász: *On uniform convergence of trigonometric series.*

This paper contains generalizations of some theorems due to Chaundy and Jolliffe, to Hardy, and to the author. The following are some of the results. The trigonometric series  $\sum_1^n b_n \sin nt$  is uniformly convergent if  $\sum_1^{2n} |b_\nu - b_{\nu+1}| = O(n^{-1})$  and if the sequence  $nb_n$  is Abel summable to zero. The power series  $\sum_1^\infty c_n z^n$  is uniformly convergent in  $|z| \leq 1$ , if  $\sum_1^{2n} |c_\nu - c_{\nu+1}| = O(n^{-1})$  and if  $\sum_1^\infty c_n$  is Abel summable. The essential part of the proof concerns the point  $z=1$ , that is  $t=0$ ; a device of the Tauberian type is employed. (Received March 18, 1944.)

150. F. A. Valentine: *Contractions in non-euclidean spaces.*

Let  $f(x)$  be a function mapping a set  $S$  in a metric space  $M$  into a set  $S'$  in a metric space  $M'$ , and suppose a contraction of the type  $\|f(x_1), f(x_2)\|' \leq \|x_1, x_2\|$  holds in  $S$  and  $S'$ . The existence of an extension of the range of definition of such a function so as to preserve a contraction depends upon  $M$  and  $M'$ . In this article the author shows *the extension exists when  $M=M'$  is the  $n$ -dimensional hyperbolic space*. The proof used is applied to a metric space which includes both the hyperbolic and the spherical cases. Hence a unification of results is also obtained. (Received February 21, 1944.)

151. S. E. Warschawski: *On conformal mapping of nearly circular regions.*

Generalizing results of L. Bieberbach (Sitzungsberichte, Berliner Akademie, 1923) and of A. R. Marchenko (Bull. Acad. Sci. U.S.S.R. 1935), the author proves the following theorem: Let  $R$  be a simply connected region with the properties: (i)  $R$  contains the origin  $w=0$  and its boundary lies in the ring  $1 \leq |w| \leq 1+\epsilon$ ,  $\epsilon$  being a fixed positive number; (ii) there exists a number  $\eta \geq \epsilon$  such that any two points  $P_1$  and  $P_2$  of  $R$  of distance less than  $\epsilon$  can be connected in  $R$  by an arc of diameter less than  $\eta$ . If  $w=f(z)$  maps the circle  $|z| < 1$  conformally onto  $R$  ( $f(0)=0, f'(0)>0$ ) then, for all  $|z| < 1$ ,  $|f(z)-z| \leq B\epsilon \log(1/\epsilon) + 4\eta$ , where  $B$  is an absolute constant. Analogous results for the derivatives of the mapping function, such as the following, are established. Let  $C$  be a simple closed curve  $\rho = \rho(\phi)$ ,  $0 \leq \phi \leq 2\pi$  ( $\rho, \phi$  polar coordinates), such that  $1 \leq \rho(\phi) \leq 1+\epsilon$ ,  $|\rho'/\rho| \leq \epsilon$ , and that  $|\rho'(\phi_2)/\rho(\phi_2) - \rho'(\phi_1)/\rho(\phi_1)| \leq \epsilon|\phi_2 - \phi_1|$ ,  $0 < \epsilon < 1$ . If  $f(z)$  (normalized as above) maps  $|z| < 1$  onto the interior of  $R$ , then, for  $|z| \leq 1$ ,  $(A(1+\epsilon^2)^{1/2})^{-1} \leq |zf'(z)/f(z)| \leq A(1+\epsilon^2)^{1/2}$  and  $|f'(z)-1| \leq 5(A\epsilon+A-1)$ , where  $A=4^\epsilon e^{\epsilon^2}$ . (Received April 1, 1944.)

#### APPLIED MATHEMATICS

152. Wilfred Kaplan and Max Dresden: *The mechanism of the condensation of gases.*

The criterion previously formulated (see abstract 49-5-158) for the condensation of a gas: namely, that condensation occurs at energy zero, when the topological structure of the energy surface changes, is further explored. It leads to a qualitative picture