TOPOLOGICAL ANALOG OF THE WEIERSTRASS DOUBLE SERIES THEOREM

G. T. WHYBURN

Since any light interior transformation of a sphere or a Riemann surface into a sphere is topologically equivalent to an analytic transformation and any non-constant analytic transformation is light and interior,[1] it might be expected in view of the Weierstrass Double Series Theorem that the limit of a uniformly convergent sequence of light interior mappings of a sphere into a sphere would be light and interior. However, this is readily seen not to be the case. For if we let r denote $|z|$ and for each $n>0$ we map the complex sphere onto itself by the function $f_n(z)$ defined by

$$\begin{aligned} w &= z && \text{for } r \geqq 2, \\ w &= z/n && \text{for } r \leqq 1, \\ w &= (r-1)z + (2-r)(z/n) && \text{for } 1 < r < 2, \end{aligned}$$

each mapping is topological, whereas the sequence $f_n(z)$ converges uniformly to the mapping $f(z)$:

$$\begin{aligned} w &= z && \text{for } r \geqq 2, \\ w &= 0 && \text{for } r \leqq 1, \\ w &= (r-1)z && \text{for } 1 < r < 2. \end{aligned}$$

The latter mapping is neither light nor interior. (Of course, analytic mappings topologically equivalent to the transformations $f_n(z)$ can be chosen so as to exhibit a wide range of behaviors, since they need only be topological mappings.)

It will be noted in this example, however, that if we factor the limit mapping $w=f(z)$ into the form $f_2f_1(z)$ where f_1 is monotone and f_2 is light,[2] *the transformation f_2 is light and interior*. This suggests the possibility, which will indeed be realized in the much more general situation where the mappings operate on an arbitrary locally connected continuum, that the limit of a uniformly convergent sequence of light interior mappings always factors into a monotone transformation followed by a light interior one. In fact, it will be shown that if the

Presented to the Society under the title *Mapping classes for locally connected continua*, November 27, 1943; received by the editors December 27, 1943.

[1] See S. Stoïlow, *Leçons sur les principes topologiques de la théorie des fonctions analytiques*, Paris, 1938, Gauthier-Villars.

[2] See, for example, the author's *Analytic topology*, Amer. Math. Soc. Colloquium Publications, New York, 1942, chap. 8. Note references therein also to S. Eilenberg.