THE TRANSFORMATION OF ČECH

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1. Introduction. The purpose of this paper is to give a simple construction of the general transformation of Čech [1, p. 192].[1]

Let the differential equations of a surface S be written in the Fubini canonical form [2, p. 123]

$$(1)\qquad \begin{aligned} x_{uu} &= \theta_u x_u + \beta x_v + px, \\ x_{vv} &= \gamma x_u + \theta_v x_v + qx, \end{aligned} \qquad \theta = \log(\beta\gamma).$$

Let the differential equation defining a conjugate net N on S be written in the form

$$(2)\qquad dv^2 - \lambda^2 du^2 = 0.$$

The ray and the associate ray intersect in *the canonical point* [3, p. 7] of N. The line joining the point x to the canonical point intersects the reciprocal of the Green-Fubini projective normal in a point whose coordinates are

$$(3)\qquad (\beta/\lambda^2)x_u - \gamma\lambda^2 x_v.$$

We shall call this point *the conjugal point* of N at x.

2. Conjugal quadrics. Let the coordinates X of a point X be written in the form

$$X = x_1 x + x_2 x_u + x_3 x_v + x_4 x_{uv}.$$

Then with properly selected unit point, (x_1, x_2, x_3, x_4) are the coordinates of X referred to the tetrahedron (x, x_u, x_v, x_{uv}). The equation of the three-parameter family of quadrics each of which has second order contact [2, p. 142] with S at x is

$$(4)\qquad x_2x_3 + x_4(-x_1 + k_2x_2 + k_3x_3 + k_4x_4) = 0.$$

The equation of any plane through the conjugal point (3) is

$$(5)\qquad x_1 - k(\gamma\lambda^2 x_2 + (\beta/\lambda^2)x_3) - 2lx_4 = 0.$$

We shall speak of this plane as *the conjugal plane* of N at x.

If we impose the condition that the polar plane of the covariant point (0, 0, 0, 1) with respect to the quadric (4) be the conjugal plane

Presented to the Society, November 27, 1943; received by the editors December 10, 1943.

[1] Numbers in brackets refer to the references cited at the end of the paper.