

to any conformal map of a sphere upon a plane lead to new characterizations of the Mercator, stereographic, and the general Lambert conical projections. Thus the only conformal map with straight scale curves is the Mercator; and the only circular cases are the stereographic and Lambert maps. (Received January 25, 1944.)

### STATISTICS AND PROBABILITY

96. Benjamin Epstein and C. W. Churchman: *On the statistics of sensitivity data.*

"Sensitivity data" is a general term for that type of experimental data for which the measurement at any point in the scale destroys the sample. The paper is a generalization of a method of treating such data due to Spearman. (C. Spearman, *The method of "right and wrong cases" (constant stimuli) without Gauss' formulae*, British Journal of Psychology vol. 2 (1908) pp. 227-242.) Formulae for the moments and their standard sampling errors are given. Certain minimization problems are also discussed. (Received January 26, 1944.)

97. E. J. Gumbel: *The observed return period.*

The theoretical return period  $T(x)$  of a value equal to, or greater than,  $x$  is defined as the inverse of the probability  $1 - F(x)$ . The question is how to calculate, for  $n$  observations, the return period  $T(x_m)$  of the  $m$ th observation  $x_m$  ( $m = 1, 2, \dots, n$ ), and especially  $T(x_n)$  of the largest observation  $x_n$  for an unlimited variate. This problem is important in probability papers where the variate is plotted as a function of the return period. Engineers use a compromise between the exceedance interval  $T(x_m) = n/(n-m)$  and the recurrence interval  $T(x_m) = n/(n-m+1)$ , namely  $T(x_m) = n/(n-m+1/2)$ . In this case  $T(x_n) = 2n$ . If, however, the probability  $F(\bar{x}_n)$  of the median  $\bar{x}_n$  of the largest value is attributed to  $x_n$ ,  $T(x_n) = 1.44n + 1/2$ . Both methods can hardly be justified. The author attributes the probability  $F(\tilde{x}_n)$  of the most probable largest value  $\tilde{x}_n$  to  $x_n$ . Then  $T(x_n)$ , as is to be expected, converges toward  $n$ , and equals  $n$  for the exponential distribution, and  $n+1$  for the logistic distribution. In the same way, the probability  $F(\tilde{x}_1)$  of the most probable smallest value  $\tilde{x}_1$  is used, for an unlimited variate, as frequency of the smallest observation  $x_1$ . The frequencies  $F(\tilde{x}_m)$  of the intermediate  $n-2$  observations are obtained by linear interpolation between  $F(\tilde{x}_1)$  and  $F(\tilde{x}_n)$ . Thus the return periods may be determined for all observations. (Received January 27, 1944.)

98. H. E. Robbins: *On the measure of a random set.*

Let  $X$ , a measurable subset of Euclidean  $n$ -dimensional space  $E$ , be a random variable (for example,  $X$  may be the set-theoretical sum of  $N$  possibly overlapping and independently chosen unit intervals on a line with a given probability distribution for their centers). Let  $m(X)$  denote the measure of  $X$ , and for any point  $x$  of  $E$  let  $p(x)$  denote the probability that  $X$  contain  $x$ . Then under very general hypotheses on  $X$  it is shown that the expected value of  $m(X)$  is equal to the integral over  $E$  of  $p(x)$ . More generally, the expected value of the  $r$ th power of  $m(X)$  is equal to the integral over  $r$ -dimensional space of the function  $p(x_1, \dots, x_r)$  = probability that  $X$  contain all the points  $x_1, \dots, x_r$ . (Received January 28, 1944.)