57. P. M. Whitman: *Lattices and equivalence relations.* Preliminary report.

It is shown that any lattice is isomorphic to a sublattice of the lattice of all equivalence relations on some set. (Received January 28, 1944.)

ANALYSIS

58. Jesse Douglas: *Separable transformations with separable inverse.*

All transformations $X=f(x)+h(y)$, $Y=g(x)+k(y)$ are found whose inverses are of the same form. Six essentially different types are obtained. If x, y are interpreted as minimal coordinates $u+iv$, $u-iv$ (and X, Y similarly), we have all harmonic transformations whose inverses are harmonic. The paper will be published in full. (Received January 15, 1944.)

59. K. O. Friedrichs: *The identity of weak and strong extensions of differential operators.*

In applying the theory of linear operators in Hilbert spaces or spaces L_p to the solution of differential equation problems, it is impossible to retain the meaning of differentiation in the ordinary sense; the concept of differential operator must be extended. Two such extensions offer themselves, a "weak" and a "strong" one, that is, the adjoint of the "formal-adjoint" and essentially the closure. The purpose of the paper is to prove the identity of these two extensions for general linear differential operators. The main tool for the proof is a certain class of smoothening operators approximating unity. They yield the identity of both extensions immediately for differential operators with constant coefficients; they are a strong enough tool to yield this identity likewise for operators with non-constant coefficients. (Received December 3, 1943.)

60. B. M. Ingersoll: *On singularities of solutions of linear partial differential equations.*

Let $U(z, \bar{z})$, $z=x+iy$, $\bar{z}=x-iy$, x, y real, be a real solution of $L(U)\equiv\Delta U+AU_x+BU_y+CU=0$, where A, B, and C are entire functions when x and y are extended to complex values. To every such solution corresponds uniquely a complex solution $u(z, \bar{z})=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}z^m\bar{z}^n$ of $L(U)=0$, with the property that $\sum_{n=0}^{\infty}A_{0n}\bar{z}^n=\pi U(0, 0)$ $\exp\left(-\int_0^{\bar{z}} a(0, \bar{z})d\bar{z}\right)$, where $a(z, \bar{z})\equiv(1/4)\{A[((z+\bar{z})/2, (z-\bar{z})/2i)]+iB[((z+\bar{z})/2, (z-\bar{z})/2i)]\}$. These solutions were introduced by Bergman (Rec. Math. (Mat. Sbornik) N. S. vol. 2 pp. 1169–1198 and Trans. Amer. Math. Soc. vol. 53 pp. 130–155) who showed that the location of the singularities of $u(z, \bar{z})$ is determined by the sequence $\{A_{m0}\}$. Employing this result the author investigates the relations between sequences $\{A_{mk}\}$, k fixed, $m=0, 1, 2, \cdots$, and the positions of singularities of $u(z, \bar{z})$. For example, using a result of Mandelbrojt (C. R. Acad. Sci. Paris, 1937, pp. 1456–1458) he determines the arguments of the singularities on the circle of convergence of $u(z, \bar{z})$ in terms of the sequence $\{A_{mk}\}$, k fixed. In the last section of the paper, using explicitly an integral representation of the complex solutions $u(z, \bar{z})$, the author investigates the real solutions $U(z, \bar{z})=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}D_{mn}z^m\bar{z}^n$ of $L(U)=0$. He constructs, in terms of $\{D_{mk}\}$, k fixed, $m=0, 1, 2, \cdots$, and some of the deriva-