ON r-REGULAR CONVERGENCE

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In his paper *On sequences and limiting sets* [1],[1] G. T. Whyburn introduced the notion of regular convergence. He showed that in the cases of 0 and 1 regular convergence (see definition below) that the limit of sequences of many simple topological sets is of the same type as the members of the sequence. It is the purpose of this paper to extend some of these results to higher dimensions. The lack of simple characterizations of the higher dimension sets (such as the n-sphere) makes the results much weaker than in the 0 and 1 dimensional cases.

It is assumed throughout the paper that all sets lie in a compact metric space. All our complexes and cycles will be non-oriented, and the Vietoris cycles and chains (V-cycles and V-chains) will have these as coordinates. The set of all points x whose distance from a set A is less than ϵ will be denoted by $U_\epsilon(A)$. Finally we shall denote the boundary of an r-dimensional complex (or V-chain) z^r by $\dot{z}^r$.

DEFINITION. *A sequence of closed sets (A_i) converging to a limit set A is said to converge r-regularly ($\rightarrow r$) if for every $\epsilon>0$ there exist numbers $\delta>0$ and $N>0$ such that, if $n>N$, any r-dimensional V-cycle in A_n of diameter less than δ is ~ 0 in a subset of A_n of diameter less than ϵ. If $A_i \rightarrow sA$ for all $s \leqq r$, we write $A_i \rightarrow \leqq rA$* [1].

DEFINITION. *A Vietoris cycle $\xi^r=(x_i^r)$ is called a projection cycle if* $\lim_{i\rightarrow\infty}$ *(point set x_i^r) $=X$ and each $x_i^r \subset X$. Clearly X is the smallest carrier* [2] *of ξ^r.*

Note. Corresponding to any cycle $\xi^r=(x_i^r)$ of a compact set F, there always exists a projection cycle $\xi_1^r \sim \xi^r$ in F. In fact if a convergent subsequence of (x_i^r) is chosen, this set can be used as the set X of the definition.

THEOREM 1. *If $A_i \rightarrow rA$, then for any $\epsilon>0$ there exist positive numbers δ and N such that if x^{r+1} is a simplex of A_i $(i>N)$ whose boundary has a V-chain realization* [3] *of diameter less than δ, then x^{r+1} has a V-chain realization of diameter less than ϵ.*

PROOF. Let δ and N be the numbers corresponding to ϵ in the definition of r-regular convergence and consider a simplex x^{r+1} of A_i $(i>N)$

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[1] Numbers in brackets refer to the bibliography at the end of the paper.