A TRANSFORMATION OF JONAS SURFACES

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It is well known that when an analytic surface S is referred to its asymptotic net (u, v) the homogeneous point coordinates $x^i(u, v)$ $(i=1, 2, 3, 4)$ of a generic point on S can then be normalized, so that they satisfy the differential equations,

$$\begin{cases} x_{uu} = \beta x_v + px, \\ x_{vv} = \gamma x_u + qx, \end{cases} \tag{1}$$

where the coefficients β, γ, p, q satisfy the conditions of integrability,

$$\begin{cases} (\beta_v + 2p)_v = (\beta\gamma)_u + \beta\gamma_u, \quad (\gamma_u + 2q)_u = (\beta\gamma)_v + \gamma\beta_v, \\ (p_v + \beta q)_v + \beta_v q = (q_u + \gamma p)_u + \gamma_u p. \end{cases} \tag{2}$$

The conjugate net Ω of S defined by

$$Cdu^2 + Ddv^2 = 0,$$

has equal point invariants when and only when[1]

$$(\log (C/D))_{uv} - (\gamma(C/D))_v + (\beta(D/C))_u = 0. \tag{3}$$

The necessary and sufficient condition that Ω should have equal tangential invariants is obtained from (3) by replacing β, γ by $-\beta$, $-\gamma$ respectively. If Ω has equal invariants, both point and tangential, then it is a Jonas net, and S then becomes a Jonas surface.[2] For a Jonas net we have thus the following relations:

$$(\log (C/D))_{uv} = 0, \qquad (\gamma(C/D))_u - (\beta(D/C))_v = 0.$$

By a suitable transformation of asymptotic parameters, leaving the asymptotic net unaltered, the above equations reduce to

$$\beta_u = \gamma_v, \qquad C = D.$$

Hence a Jonas net on a Jonas surface S may be represented by the equation

$$du^2 - dv^2 = 0, \tag{4}$$

and the surface is characterized by

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[1] Cf. G. Fubini-E. Čech, *Geometria Proiettiva Differenziale*, vol. 1, Bologna, Zanichelli, 1927, p. 105.

[2] Cf. Fubini-Čech, ibid. p. 106.