

TIBOR RADÓ

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SOLUTION OF THE “PROBLÈME DES MÉNAGES”

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The *problème des ménages* asks for the number of ways of seating n husbands and n wives at a circular table, men alternating with women, so that no husband sits next to his wife. Despite the considerable literature devoted to this problem (cf. the appended bibliography), the following simple solution seems to have been missed.

It is convenient first to solve two preliminary problems, perhaps of some interest in themselves.

LEMMA 1. *The number of ways of selecting k objects, no two consecutive, from n objects arrayed in a row is ${}_{n-k+1}C_k$.*

Let $f(n, k)$ be the desired number. We split the selections into two subsets: those which include the last of the n objects and those which do not. The former are $f(n-2, k-1)$ in number (since further selection of the second last object is forbidden); the latter are $f(n-1, k)$ in number. Hence

$$f(n, k) = f(n-1, k) + f(n-2, k-1),$$

and, combining this with $f(n, 1) = n$, we readily prove by induction that $f(n, k) = {}_{n-k+1}C_k$.

LEMMA 2. *The number of ways of selecting k objects, no two consecutive, from n objects arrayed in a circle is ${}_{n-k}C_k n / (n-k)$.*

This differs from the preceding problem only in the imposition of the further restriction that no selection is to include both the first and last objects; and the number of such selections which are otherwise acceptable is $f(n-4, k-2)$. Hence the desired result is $f(n, k) - f(n-4, k-2) = {}_{n-k}C_k n / (n-k)$.