DECOMPOSITIONS OF A T_1 SPACE

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Introduction. Several authors have proved theorems of the type: The "structure" of a certain class of transformations defined on a "suitable" space A to a fixed "suitable" space B "determines" the space A.

As examples, we have:

Banach [3, p. 170, see also 6, 7, 13]:[1] The Banach space of all real, continuous functions defined on a compact metric space A "determines" A.

Eidelheit [5, see also 2, 10, 11]: The ring of all bounded operators on a real Banach space A "determines" A.

In the present paper, we prove an analogous theorem (Theorem 2.5). Intuitively, it says that a T_1 space A is "determined" by a rather weak ordered system structure of the collection of all continuous mappings of A onto an arbitrary (variable) T_1 space B. More exactly, it states: If two T_1 spaces A, B are such that the ordered system of upper semi-continuous decompositions of A is isomorphic to that of B, then A and B are homeomorphic.

In §1 we give a discussion of ordered systems which is sufficient for our purposes. In §2 we prove the theorem mentioned above. In §3 we characterize separation and connectedness properties of a T_1 space in terms of order properties of its upper semi-continuous decompositions. In §4 we discuss compactness properties of the space and their relations to order properties of the decompositions, and in §5 we give some examples and counter examples.

1. Ordered systems.[2]

We assume that the reader is fairly familiar with the nomenclature of ordered systems and lattices.

An *ordered system* is a collection, M, of elements, D, with an ordering defined in M. That is, there is given a binary relation, $>$, which is defined for some pairs of elements in M, and which is transitive, reflexive, and proper.

We use *less than* and *greater than* to refer to the ordering in M; *contains* will be used only in the point set sense.

Most of the ordered systems we consider will be directed and will contain atoms. Throughout the paper, we use the letters a, b, c for

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[1] Numbers in square brackets refer to the bibliography at the end of the paper.

[2] For a more complete discussion, see [4, 12, 14].