ON HARMONIC AND ANALYTIC FUNCTIONS

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If we study the behavior of a harmonic function on the boundary of the unit circle along an arc $\alpha<\theta<\beta$, it is sometimes of advantage, if the function behaves in the simplest possible way outside this arc. This problem of *isolating* the singular arc can easily be solved for a harmonic function which is bounded in the unit circle. For such a function can be expressed by means of a Poisson integral

$$u(r, \theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1 - r^2}{1 - 2r\cos(\theta - \vartheta) + r^2} u(1, \vartheta)d\vartheta,$$

and then

$$v(r, \theta) = \frac{1}{2\pi}\int_{\alpha}^{\beta} \frac{1 - r^2}{1 - 2r\cos(\theta - \vartheta) + r^2} u(1, \vartheta)d\vartheta$$

is known *to behave in the same way* as $u(r, \theta)$ in the neighborhood of the arc (α, β)—in fact the difference of the two functions tends uniformly to zero inside the arc—and $v(r, \theta)$ can be extended so as to make it harmonic and equal to zero on the rest of the circumference.

It is equally easy to solve the problem for a harmonic function $u(r, \theta)$ which is $O(1/(1-r)^n)$ near the circumference and is, therefore, the $(n+2)$nd derivative of a harmonic function, bounded in $r \leqq 1$.

The purpose of this paper is to show that the problem can be solved for any function harmonic in $r<1$. The result can be generalized to any domain which can be represented conformally on the unit circle.

THEOREM. *If $u(r, \theta)$ is a function, harmonic in the unit circle, then, given the arc $(r=1, \alpha<\theta<\beta)$, there is a function $v(r, \theta)$ harmonic in $r<1$, such that $u(r, \theta)-v(r, \theta)$ can be extended across the arc (α, β) so as to make it harmonic and zero on the arc; and $v(r, \theta)$ is harmonic and zero along the rest of the circumference.*

PROOF. (i) If

$$u(r, \theta) = \sum_1^\infty r^n(a_n \cos n\theta + b_n \sin n\theta),$$

we can find a nonincreasing sequence $\epsilon_n \to 0$, such that

$$(1) \qquad |a_n|, \ |b_n| \leqq ((1/2)e^{\epsilon_n n} - 2)/n^2.$$

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