ZEROS AND POLES OF FUNCTIONS DEFINED BY TAYLOR SERIES

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1. **Introduction.** With the objective of providing a straightforward numerical method for the determination of poles and zeros of functions defined by Taylor series this note reexamines Hadamard's solution of this problem, which is found in his classical thesis.[1] The best known part of Hadamard's solution is the criterion which enables one to determine the meromorphic character of the expanded function and the total number of poles on the circle of convergence. But this solution also includes a method of determining these poles as functions of the Taylor coefficients, and Hadamard himself intimated that his results should prove useful in the numerical evaluation of poles and zeros. However, it seems that, as a device in numerical analysis, his method has attracted much less attention than it deserves. This may be due to the fact that Hadamard's criterion for the number of poles employs limits superior, which are impractical for numerical work.

In this paper no use is made of limits superior, and the number of poles on the circle of convergence is ascertained by the process of evaluating their affixes. Besides determining the polynomial whose zeros are all the poles of the expanded function on the circle of convergence with their proper multiplicities, the paper also determines the polynomial whose zeros are the different poles of highest order only. These results are based on an identity and an inequality for persymmetric determinants involving successive Taylor coefficients of rational and meromorphic functions, which seem to be new, and may also prove useful in other applications.

2. **A formula for persymmetric determinants.** We first are going to establish an identity for persymmetric determinants of the form

$$(1)\qquad d_n^{(m)} = \begin{vmatrix} c_{n+1} & c_{n+2} & \cdots & c_{n+m} \\ c_{n+2} & c_{n+3} & \cdots & c_{n+m+1} \\ \cdot & \cdot & \cdots & \cdot \\ c_{n+m} & c_{n+m+1} & \cdots & c_{n+2m-1} \end{vmatrix}, \qquad n = -1, 0, 1, \cdots,$$

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[1] J. Hadamard, *Essai sur l'étude des fonctions données par leur développement de Taylor*, J. Math. Pures Appl. (4) vol. 8 (1892) pp. 101–186.