NEW SYSTEMS OF HYPERGEODESICS DEFINED ON A SURFACE

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Introduction. Let a non-ruled surface S be referred to its asymptotic net as parametric. As a point P_y moves along a curve C_λ of S, the tangents at P_y to the u- and v-asymptotic curves of S describe two ruled surfaces R_λ^u and R_λ^v, respectively. Let S_ρ and S_σ denote arbitrary transversal surfaces of the congruences of u- and v-tangents of S, respectively. The purpose of the present paper is to introduce and study systems of curves of S which will be called ρ- and σ-*tangeodesics.*

DEFINITION. *A curve C_λ of S whose associated ruled surface R_λ^u intersects the surface S_ρ in an asymptotic curve of R_λ^u is a ρ-tangeodesic of S. Similarly, a curve C_λ of S whose associated ruled surface R_λ^v intersects S_σ in an asymptotic curve of R_λ^v is a σ-tangeodesic of S.*

The ρ- and σ-*tangeodesics* of S at P_y are found to be associated in remarkable manners with the *edges of Green*, the *directrices of Wilczynski*, and the *projective normal of Fubini*. In fact, a new geometric characterization is obtained for each of these lines.

1. **Tangeodesics.** If the parametric net on a non-ruled surface S is the asymptotic net, the homogeneous projective coordinates $y^{(i)}(u, v)$ $(i=1, 2, 3, 4)$ of a general point P_y of S are solutions of a system of differential equations which may be assumed to be reduced to Wilczynski's canonical form

$$(1.1) \qquad y_{uu} + 2by_v + fy = 0, \qquad y_{vv} + 2a'y_u + gy = 0.$$

The homogeneous coordinates of points ρ, σ on arbitrarily selected transversal surfaces S_ρ and S_σ of the congruences of u- and v-tangents of S are given by the vector forms

$$(1.2) \qquad \rho = y_u - \beta y, \qquad \sigma = y_v - \alpha y,$$

wherein β, α are arbitrary analytic functions of u, v.

Let l denote the line joining ρ, σ and let l' denote its reciprocal at P_y. The line l' joins the points P_y and z where z is given by

$$(1.3) \qquad z = y_{uv} - \alpha y_u - \beta y_v$$

in which β and α are the functions in (1.2). The line l, according to Green's classification, is a line of the first kind and generates a con-

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