ON A NEW APPLICATION OF JACOBI POLYNOMIALS IN CONNECTION WITH THE MEAN VALUE THEOREM

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Let us consider the classical theorem of mean value, which states that

$$(1) \qquad U(\xi) = \frac{1}{b-a}\int_a^b U(x)dx$$

or

$$(1') \qquad F'(\xi) = \frac{1}{b-a}[F(b) - F(a)]$$

can be satisfied by at least one value of ξ inside[1] the interval (a, b).

In the general case we can add no further precision concerning the position of the value ξ inside the interval (a, b). But if we consider only functions $U(x)$ belonging to a definite class of functions, we can, sometimes, give a more precise determination for this value ξ. We can, in particular, for some classes of functions, determine intervals (a', b'), concentric to (a, b), with

$$(2) \qquad b' - a' = \theta(b-a), \qquad 0 \leqq \theta \leqq 1,$$

and such that (1) holds for at least one value ξ inside (a', b'), for every function $U(x)$ belonging to the class considered and for every interval (a, b) for which the classical mean value theorem holds. The smallest number θ which has the above mentioned property for a given class of functions is called its "contraction factor." It results from this definition that the value of the contraction factor depends only on the class of functions considered and is independent of all other factors, such as the interval (a, b), and so on.

If we replace the equation (1) by (1') and repeat the foregoing literally, we define in exactly the same way the contraction factors for classes of functions $F(x)$.

The existence of a contraction factor for certain classes of functions, particularly for polynomials of a real variable, has been proved by Paul Montel.[2] The value of θ as a function of the degree n of the polynomials considered was found independently and almost at the

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[1] The expression "inside" in this paper means: within or at the ends.

[2] P. Montel, Bull. Soc. Math. France vol. 58 (1930) pp. 105–126. See also D. Pompeiu, Annales Scientifiques de l'Université de Jassy vol. 15 (1929) p. 335.