GROUPS TRANSITIVE ON THE n-DIMENSIONAL TORUS

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In this note we denote by G a compact connected Lie group. We shall be interested in the situation where G acts as a topological transformation group[2] on a space E. Such a group is called effective if the identity is the only element of G which leaves every point of E fixed. If G is transitive on E, that is, for any two points x and y of E there is an element g of G such that $g(x)=y$, then E is called a homogeneous space or a coset space of G. Our purpose is to prove the following theorem:

THEOREM. *If a compact connected Lie group G is transitive and effective on a space E homeomorphic with an n-dimensional torus (topological product of n circles), then G is isomorphic with the n-dimensional toral group T_n (direct product of n circle groups) and no element of G except the identity leaves any point of E fixed.*

We use a method of proof which has some similarity to a method we have used in studying groups transitive on spheres.[3]

Let H' be a compact, connected, simply connected Lie group, let T_l be an l-dimensional toral group, and let N be a finite normal subgroup of the direct product $H'\times T_l$ such that G is continuously isomorphic to the factor-group $(H'\times T_l)/N$.[4] Let H' go into H by the homomorphism obtained by factoring with respect to N and let T_l go into K. The group K is also an l-dimensional toral group, and H and K are subgroups of G which span G or generate G. The elements of H commute with the elements of K, in fact K is a central subgroup of G.

Let x be an arbitrarily chosen point of E and let H_x, K_x, and G_x be, respectively, the subgroups of H, K, and G which leave x fixed. Let K^x be the subgroup of K consisting of those elements k such that $k(x)$ is in the orbit $H(x)$. The orbit $K^x(x)$ is the intersection of $H(x)$ and $K(x)$. It can be seen that if $y=g(x)$ then $K_y=gK_xg^{-1}$ and $H_y=gH_xg^{-1}$. Since K is a central subgroup we see that $K_y=K_x$.

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[2] For the theory of topological groups and Lie groups needed see Pontrjagin, *Topological groups*, Princeton 1939. For definitions and results concerning topological transformation groups see Zippin, *Transformation groups*, Lectures in Topology, Ann Arbor 1941 pp. 191–221.

[3] See a paper by us which is forthcoming.

[4] For the existence of these groups see Pontrjagin, loc. cit. pp. 282–285. The group H' is the direct product of the simple Lie groups there mentioned.