THE BETTI GROUPS OF SYMMETRIC AND CYCLIC PRODUCTS

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1. **Introduction.** Consider a finite complex K and a group of permutations of n elements $G=\{G_\lambda\}$, $\lambda=1, \cdots, N$. To define *the product k^n of K with respect to G*, $n=2, 3, \cdots$, we consider an ordered set of n complexes $K_1, \cdots, K_n$ each homeomorphic to K; here as throughout the paper we do not distinguish between a complex and a geometric realization of the complex. A point p of the topological product $K^n=K_1\times \cdots \times K_n$ can be represented by the sequence of points $p_1, \cdots, p_n$, $p_i \in K_i$. Each function $G_\lambda(p)$, $\lambda=1, \cdots, N$, gives a homeomorphism of K^n upon itself. We identify each point $p\in K^n$ with all its transforms $G_\lambda(p)$, $\lambda=1, \cdots, N$. The resulting continuous image of K^n is k^n. If G is the symmetric group or the cyclic group of permutations of n elements, the product k^n is called the *n-fold symmetric product* or the *n-fold cyclic product of K*, respectively.

In this paper we study the integral cohomology groups of k^n. Our Theorem 1 gives a convenient method for calculating these groups when G is given. The method is used to construct the cohomology groups when G is either symmetric or cyclic.

The method of this paper differs from that of the earlier papers [**3**] and [**5**] of the references at the end of this paper in the following way. All treatments consider Richardson's simplicial transformation Λ of K^n upon k^n. But Richardson and Walker use Λ to determine a transformation of cycles of K^n into cycles of k^n, while this paper considers the natural transformation of cocycles of k^n into cocycles of K^n. The earlier correspondence of cycles is not (1-1), but the present correspondence of cocycles is (1-1). This fact enables us to get new results.

2. **The general theorem.** By definition k^n is obtained by identifying points of K^n. This identification gives a continuous transformation Λ of K^n upon k^n. Richardson has shown[1] that K^n and k^n can be subdivided into simplicial complexes and the simplexes of these complexes so oriented that Λ is simplicial, G_λ is simplicial, $\lambda=1, \cdots, N$, and for any oriented simplex x of K^n

$$\Lambda x = \Lambda G_\lambda x, \qquad \lambda = 1, \cdots, N. \tag{1}$$

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[1] See [**3**, §5].