A REMARK ON ALGEBRAS OF MATRICES

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1. **Introduction.** Let $\mathfrak{A}$ denote a matrix algebra, with unit element, over an algebraically closed field K. We shall assume that $\mathfrak{A}$ is in reduced form, that is, that $\mathfrak{A}$ is exhibited with only zeros above the main diagonal, with irreducible constituents of $\mathfrak{A}$ in the main diagonal, and that $\mathfrak{A}$ is expressible as the direct sum of its radical and a semisimple subalgebra which latter has nonzero components only in the irreducible constituents of $\mathfrak{A}$:

$$\mathfrak{A} = \begin{pmatrix} \mathfrak{C}_{11} & \cdot & \cdots & \cdot \\ \mathfrak{C}_{21} & \mathfrak{C}_{22} & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mathfrak{C}_{t1} & \mathfrak{C}_{t2} & \cdots & \mathfrak{C}_{tt} \end{pmatrix}, \tag{1}$$

the $\mathfrak{C}_{ii}$ denoting irreducible constituents; further $\mathfrak{A}=\mathfrak{A}^*+\mathfrak{N}$ where $\mathfrak{N}$ is the radical of $\mathfrak{A}$ and

$$\mathfrak{N} = \begin{pmatrix} 0 & \cdot & \cdots & \cdot \\ \mathfrak{C}_{21} & 0 & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mathfrak{C}_{t1} & \mathfrak{C}_{t2} & \cdots & 0 \end{pmatrix}, \qquad \mathfrak{A}^* = \begin{pmatrix} \mathfrak{C}_{11} & \cdot & \cdots & \cdot \\ 0 & \mathfrak{C}_{22} & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & \mathfrak{C}_{tt} \end{pmatrix}. \tag{2}$$

As a part of $\mathfrak{A}$, $\mathfrak{C}_{ij}$ forms an additive group or module of matrices upon which $\mathfrak{A}$, itself considered as a module, is homomorphically mapped. We shall consider $\mathfrak{C}_{ij}$ as a matrix module with $\mathfrak{A}$ as both left and right operator system. For a matrix A of $\mathfrak{A}$, we shall use the notation $C_{ij}(A)$, $(j \leqq i,\ i=1, 2, \cdots, t)$, to denote the parts of A,

$$A = \begin{pmatrix} C_{11}(A) & \cdot & \cdots & 0 \\ C_{21}(A) & C_{22}(A) & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ C_{t1}(A) & \cdot & \cdots & C_{tt}(A) \end{pmatrix}. \tag{3}$$

Let B be any element of $\mathfrak{A}$, and let B^* be the component of B in the semisimple subalgebra $\mathfrak{A}^*$. We define B as a left and as a right operator of $C_{ij}(A)$ by the relations below, using $\circ$ to distinguish this operation from ordinary matrix multiplication

Received by the editors June 15, 1942.