NOTES ON THE BERTINI INVOLUTION

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1. **Introduction.** Given a pencil of plane cubic curves

$$\lambda w(x) + \mu w'(x) = 0 \tag{1}$$

with the vertices of the reference triangle among its base points. Arranged as to (0, 0, 1) the equations may be written

$$w(x) = x_3^2 u_1 + x_3 u_2 + u_3,$$
$$w'(x) = x_3^2 u_1' + x_3 u_2' + u_3',$$

with

$$u_1 = a_1 x_1 + a_2 x_2, \qquad u_1' = a_1' x_1 + a_2' x_2,$$
$$u_2 = b_1 x_1^2 + b_2 x_1 x_2 + b_3 x_2^2, \quad u_2',$$
$$u_3 = c_1 x_1^2 x_2 + c_2 x_1 x_2^2, \quad u_3',$$

and $a_i, a_i', b_i, b_i', c_i, c_i'$ generic constants.

A point y of the plane fixes the curve of the pencil (1) passing through it, hence

$$w(x)w'(y) - w'(x)w(y) = 0, \tag{2}$$

which may be written in the form

$$\begin{aligned} W_3(x) = {} & x_3(A_1x_1 + A_2x_2) + x_3(B_1x_1^2 + B_2x_1x_2 + B_3x_2^2) \\ & + C_1x_1^2x_2 + C_2x_1x_2^2 = 0 \end{aligned} \tag{3}$$

in which $A_i = a_i w'(y) - a_i' w(y)$, and similarly for B_i and C_i. The tangent to $W_3(x) = 0$ at (0, 0, 1) is

$$A_1x_1 + A_2x_2 = 0, \tag{4}$$

which meets the curve again at $R = (r_1, r_2, r_3)$,

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[1] Miss Moody, Ph.D. Cornell University, an instructor in mathematics at Pennsylvania State College, was killed in an automobile accident April 11, 1941. I had suggested that she compare my cumbersome method of derivation of the equations of this transformation (Amer. J. Math. vol. 33 (1911) pp. 327–336) with that of employing a pencil of cubic curves. The following notes were found among her posthumous papers sent me recently. The equations of the Bertini involution are simpler than those previously known, and other properties found may be extended by others.

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