INTERSECTIONS OF CONTRACTIBLE POLYHEDRA

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Let the finite polyhedron C be expressed as a sum of two polyhedra, A and B. Using the Mayer-Vietoris formula, it is easy to prove the following two propositions:

(1) *If $A \cap B$, the intersection of A and B, is acyclic,[2] then C is acyclic if and only if both A and B are acyclic.*

(2) *If A, B, and C are all acyclic, so is $A \cap B$.*

Aronszajn and Borsuk have shown[3] that proposition (1) is also true if acyclic is replaced by contractible.[4] They left open the question as to whether or not this is true for proposition (2). We show by means of an example that it is not true. The example is constructed as follows:

Let P be a Poincaré sphere, that is, a 3-dimensional polyhedron with the homology groups of a 3-sphere and with a non-vanishing fundamental group. Let K be the polyhedron obtained by removing an open 3-simplex from P. It is easy to see that K is acyclic and that its fundamental group is the same as that of P, so that K is not contractible.

Let C be the join[5] of K with two points, q and q'. Denote the join of K with q by A, and the join of K with q' by B. Let $C = A \cup B$ the sum of A and B. Then $A \cap B = K$. It is clear that A can be continuously deformed into the point q, and B into q', so both these polyhedra are contractible. Hence it remains to show only that C is contractible. By proposition (1), C is acyclic. Hurewicz has shown[6] that if the fundamental group of an acyclic polyhedron vanishes,

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[2] A polyhedron is said to be acyclic if all its homology groups vanish for all coefficient groups.

[3] N. Aronszajn, and K. Borsuk, *Sur la somme et le produit combinatoire des retracts absolus*, Fund. Math. vol. 18 (1932) pp. 193–197.

[4] A set is said to be contractible if it can be deformed over itself into a point. A contractible polyhedron is acyclic.

[5] The join of two sets, X and Y, is the collection of line segments joining each point of X with each point of Y and disjoint except for their common end points. If X and Y are polyhedra, so is their join.

[6] W. Hurewicz, *Beiträge zur Topologie der Deformationen*, Neder. Akad. Wetensch. vol. 38 (1935) pp. 521–528, Theorem IV.