EXPANSIONS OF QUADRATIC FORMS

RUFUS OLDENBURGER

1. **The problem.** A quadratic form Q with coefficients in a field K, whose characteristic is different from 2, is usually given as a linear combination

$$\sum_{i=1}^{n} a_{ij}x_i x_j \tag{1}$$

of products $\{x_i x_j\}$, where (a_{ij}) is symmetric. The sum (1) is one of the type

$$\sum_{i=1}^{\tau} L_i M_i, \tag{2}$$

where the L's and M's are linear forms. In general the decomposition (1) is not the most economical way of writing Q as a sum of the type (2) in the sense that τ is a minimum for Q. In treating algebras associated with quadratic forms E. Witt[1] showed that the form Q is equivalent under a nonsingular linear transformation to a decomposition

$$\sum_{i=1}^{\sigma} y_i z_i + \sum_{i=1}^{r-2\sigma} \nu_i u_i^2, \tag{3}$$

where the last sum is a nonzero form, and r is the rank of Q. In the present paper we shall show that the minimum τ for Q is $r-\sigma$. Thus this minimum τ is determined by the rank r and the "characteristic" σ of Q. This characteristic[2] is the maximum number σ of linearly independent linear forms $L_1, \cdots, L_\sigma$ such that the rank of $Q+\lambda_1 L_1^2+ \cdots +\lambda_\sigma L_\sigma^2$ is the same as the rank of Q for all values of the λ's. The form Q has characteristic σ *if and only if* Q has the *canonical splitting* $G+H$, where G has characteristic σ and rank 2σ, while H has characteristic 0 and rank $r-2\sigma$. The form G has a decomposition (2) with $\tau=\sigma$. The decomposition (3) is one such that the first sum is a form G of the type described and the other a form H. Thus it will be proved

Presented to the Society, April 18, 1942; received by the editors April 29, 1942.

[1] E. Witt, *Theorie der Quadratischen Formen in beliebigen Körpern*, J. Reine Angew. Math. vol. 176 (1937) p. 35.

[2] Rufus Oldenburger, *The index of a quadratic form for an arbitrary field*, Bull. Amer. Math. Soc. abstract 48-5-162.