$$5p_1 + \ \ 2p_3 + 7p_4 = m + 1, \qquad 5q_1 + \ \ 2q_3 + 7q_4 = n.$$

A solution of these equations is $p_1=4$, $p_2=3$, $p_3=11$, $p_4=7$, $p_5=1$, $q_1=4$, $q_2=3$, $q_3=11$, $q_4=7$, $q_5=3$. Hence a solution of (12) is $x=\alpha s^4t^4$, $y=\beta s^3t^3$, $u=\lambda s^{11}t^{11}$, $v=\mu s^7t^7$, $w=\nu st^3$ where $s=a\alpha^3\lambda^7\nu+b\beta^4\mu^{11}\nu$, $t=c\alpha^5\lambda^2\mu^7$.

If $x=x'$, $y=y'$, $u=u'$, $v=v'$, $w=w'$ is a given solution of (12) and the choice $\alpha=x'$, $\beta=y'$, $\lambda=u'$, $\mu=v'$, $\nu=w'$ is made then $s=t$ and the solution becomes $x=x't^8$, $y=y't^6$, $u=u't^{22}$, $v=v't^{14}$, $w=w't^4$ which is equivalent to the given solution provided $t\neq 0$.

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VECTOR ANALOGUES OF MORERA'S THEOREM

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Let the vector

$$X \equiv X(x_1, x_2, x_3) \equiv X(x) \equiv X_1 i + X_2 j + X_3 k$$

be defined and continuous in the domain (non-null connected open set) D. Consider the mean-value vector

$$X^{(\rho)}(x) \equiv \frac{1}{|V_\rho|}\int_{V_\rho} X(x+\xi)dV, \tag{1}$$

where V_ρ denotes the sphere

$$\xi_1^2 + \xi_2^2 + \xi_3^2 < \rho^2,$$

and $|V_\rho|$ its volume,

$$|V_\rho| \equiv 4\pi\rho^3/3.$$

The vector (1) can be defined thus for only a part D_ρ of D, but this is of no consequence since ρ is arbitrarily small.

Since $X(x)$ is continuous, it follows that $X^{(\rho)}(x)$ has continuous partial derivatives of the first order; these are given by

$$\frac{\partial}{\partial x_p} X^{(\rho)}(x) = \frac{1}{|V_\rho|}\int_{S_\rho} X(x+\rho\alpha)\alpha_p d\sigma, \tag{2}$$

where S_ρ denotes the surface of V_ρ and α_1, α_2, α_3 are the components of the unit vector along the outer normal to S_ρ.

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