ON MAJORANTS OF SUBHARMONIC AND ANALYTIC FUNCTIONS

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This paper represents a different approach to a whole group of problems connected with majorants of subharmonic functions. The same method has been used previously in order to prove a generalization of the Phragmén-Lindelöf theorem.[1] It seems that the best approach is to prove first Lemma 4, and then the most important results are easily deducible. Corollary 6 is a generalization of a result of N. Levinson.[2] His theorem has made me realize the importance of these results.

LEMMA 1. *If* (i) $0<f(x)\leqq 1$ *and* (ii) $\int_a^b \log f(x)\cdot dx$ *is finite, then*

$$\int_a^b \log\left|\int_\xi^x f(y)dy\right| dx \tag{1}$$

is a continuous function of ξ *in* (a, b).

We first suppose that $f(x)$ is non-decreasing and that $(0, 1)=(a, b)$. We get

$$\int_0^x f(y)dy > \int_{x/2}^x f(y)dy \geqq (x/2)f(x/2).$$

Hence

$$\begin{aligned}\int_0^1 \log\left(\int_0^x f(y)dy\right)dx &> \int_0^1 \log\,(x/2)dx + \int_0^1 \log f(x/2)dx \\ &> 2\int_0^1 \log x\cdot dx + 2\int_0^1 \log f(x)dx \\ &= -2+2\int_0^1 \log f(x)\cdot dx.\end{aligned} \tag{2}$$

If $f(x)$ is replaced by $f(a+(b-a)x)$, we obtain

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[1] Cf. the end of this paper and Journal of the London Mathematical Society, vol. 14 (1939), p. 208.

[2] *Gap and Density Theorems*, American Mathematical Society Colloquium Publications, vol. 26, 1940, p. 127, Theorem 43.