ON AN INEQUALITY OF SEIDEL AND WALSH

LYNN H. LOOMIS

Introduction. In a recent paper[1] Seidel and Walsh introduced the following concepts.

Let R be a Riemann surface (configuration) lying over the w-plane, and let C_p be a simply-connected region of R having the following properties:

(a) C_p contains precisely p points (counted according to branch-point multiplicity) lying over some point of the w-plane.

(b) C_p lies over the circle $|w-w_0|<r$, and the boundary of C_p lies over the circumference $|w-w_0|=r$.

It follows that C_p contains precisely p points lying over every point of $|w-w_0|<r$, and in particular, p points $\bar{w}_i$ lying over w_0. Seidel and Walsh name such a region a *p-sheeted circle with centers $\bar{w}_i$ and radius r.* Given a point $\bar{w}_0$ of R, let r_p be the radius of the largest p-sheeted circle in R with center $\bar{w}_0$; if none exists, let $r_p=0$. We then define the *radius of p-valence of R at $\bar{w}_0$*, $D_p(\bar{w}_0)$, as the maximum of the r_n for $n \leqq p$.

Let $w=f(z)=a_1z+\cdots+a_pz^p+a_{p+1}z^{p+1}+\cdots$ be analytic in the unit circle $|z|<1$ with $|f(z)|<M$, and let the Riemann surface R be the image of $|z|<1$ under $w=f(z)$. Let $\bar{w}_0$ be the image of $z=0$; $\bar{w}_0$ lies over $w=0$. Seidel and Walsh establish the following relation between the first p coefficients of $f(z)$ and the radius of p-valence, $D_p(\bar{w}_0)$, of R at $\bar{w}_0$.

There exist two constants, λ_p depending only on p, and Λ_p depending on p and M, such that

$$\lambda_p D_p(\bar{w}_0) \leqq \sum_{n=1}^{p} |a_n| \leqq \Lambda_p [D_p(\bar{w}_0)]^{2^{-p}}. \tag{1}$$

Seidel and Walsh find for Λ_p the value

$$\Lambda_p = 24pM^r, \qquad r = 1 - 2^{-p}.$$

In this note we prove the following two statements concerning the inequalities (1).

A. *The exponent 2^{-p} may be replaced by $1/(p+1)$ and this exponent is the best possible (for $D_p \to 0$).*

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[1] W. Seidel and J. L. Walsh, *On the derivatives of functions analytic in the unit circle and their radii of univalence and of p-valence*, Transactions of this Society, vol. 52 (1942), pp. 129–216.