GENERATORS OF PERMUTATION GROUPS SIMPLY ISOMORPHIC WITH $LF(2, p^n)$

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It is well known that the group $LF(2, p^n)$ of linear fractional transformations of determinant unity in the $GF[p^n]$ can be represented as a permutation group G of degree p^n+1. The purpose of this note is to show that the generators of G follow from a slight extension of an argument used in a recent paper.[1]

We obtain a representation of the abstract group L simply isomorphic with the special linear homogeneous group $SLH(2, p^n)$ by means of the cosets K and KTS_λ, where λ ranges over the p^n marks of the field $u_0(=0)$, $u_1, \cdots, u_m$, $(m=p^n-1)$. Let $k_\infty=K$ and $k_{u_i}=KTS_{u_i}$ for $i=0, 1, \cdots, m$.

If ρ is any mark, $KS_\rho=K$ and $KTS_\lambda \cdot S_\rho=KTS_{\lambda+\rho}$, so that to S_ρ there corresponds the permutation

$$(1) \qquad s_\rho = \begin{pmatrix} k_\infty & k_0 & k_{u_1} & \cdots & k_{u_m} \\ k_\infty & k_\rho & k_{u_1+\rho} & \cdots & k_{u_m+\rho} \end{pmatrix}.$$

If $\lambda \neq 0$, $KTS_\lambda T=KTS_{-\lambda^{-1}}$. Further, $KTS_0T=K$, so that to T there corresponds the permutation

$$(2) \qquad t = (k_0 k_\infty \cdot k_{u_1} k_{-u_1^{-1}} \cdots k_{u_m} k_{-u_m^{-1}}).$$

Hence L has a $(d, 1)$ isomorphism with (s_ρ, t), where d is the order of a subgroup of K which is invariant in L. The quotient group (s_ρ, t) is simply isomorphic[2] with $LF(2, p^n)$ and is of order $p^n(p^{2n}-1)/d$, where $d=2$ or 1 according as $p>2$ or $p=2$.

THEOREM. *A permutation group simply isomorphic with the group $LF(2, p^n)$ of linear fractional transformations of determinant unity in the $GF[p^n]$ is generated by* (1) *and* (2), *where ρ ranges over an independent set of additive generators of the field.*

COROLLARY.[3] *A permutation group simply isomorphic with the group $LF(2, p)$ is generated by $(k_0k_1k_2 \cdots k_{p-1})$ and $(k_0k_\infty \cdot k_1k_{i_1} \cdot k_2k_{i_2} \cdots)$, where $ji_j \equiv -1 \pmod p$.*

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[1] *A note on the special linear homogeneous group $SLH(2, p^n)$*, this Bulletin, vol. 47 (1941), pp. 629–632. The notation and results of this paper are assumed above.

[2] L. E. Dickson, *Linear Groups with an Exposition of the Galois Field Theory*, pp. 87–88.

[3] Compare with $x'=x+1$ and $x'=-1/x$, which generate $LF(2, p)$.