NON-INVOLUTORIAL SPACE TRANSFORMATIONS ASSOCIATED WITH A LINEAR CONGRUENCE

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1. **Introduction.** Vogt[1] has studied, using synthetic methods, the space transformations associated with a linear congruence and with a bundle of lines. In the present paper the non-involutorial transformations associated with these configurations are found analytically, and by an entirely different geometric process. It will be shown that the general transformation associated with a linear congruence so obtained differs from that of Vogt in one important respect, having one more fundamental curve in each of the projectively related spaces. The transformations in the planes containing the directrices of the congruence are also shown.

Given the directrices r and s of the congruence and two projective pencils of surfaces

$$|F_{m+n+1}| : r^m s^n g_{2m+2n+2mn+1}$$

and

$$|F'_{m'+n'+1}| : r^{m'} s^{n'} g_{2m'+2n'+2m'n'+1}.$$

Through a generic point $P(y)$ there passes a single F of $|F|$. The unique line t of the congruence through $P(y)$ meets the associated F' of $|F'|$ in one residual point $P'(x)$, the image of $P(y)$ under the transformation thus defined. The residual base curves of $|F|$ and $|F'|$, other than r and s, have been denoted by g and g', respectively. Through a point $O_{g'}$ on g' there is a unique line t' of the congruence, this line lying upon one surface of $|F'|$. The associated surface of $|F|$ meets t' in a point $\overline{P}$ which generates a curve $\bar{g}$. Similarly, beginning with a point O_g on g, a point $\overline{P}'$ generating a curve $\bar{g}'$ is found. It will be shown that r, s, g, g', $\bar{g}$, $\bar{g}'$ are fundamental curves of the transformation.

2. **Equations of the transformation.** Let us take the equations of the directrices r and s, respectively, as

$$(1) \qquad x_1 = x_2 = 0, \qquad x_3 = x_4 = 0,$$

and the pencils of surfaces

$$(2) \qquad |F_{m+n+1}| \equiv U - \lambda \overline{U} = 0, \quad |F'_{m'+n'+1}| \equiv U' - \lambda \overline{U}' = 0$$

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[1] Vogt, *Zentrale und windschiefe Raum-Verwandtschaften*, Jahresbericht der Schlesischen Gesellschaft für Vaterlanddische Kultur, class 84, 1906, pp. 8–16.