

**PROJECTIONS OF THE PRIME-POWER ABELIAN GROUP
OF ORDER p^m AND TYPE $(m-1, 1)$**

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1. **Introduction.** A function f of the subgroups of the group G upon the subgroups of the group H is called a projectivity of G upon H ($f(G) = H$) if the following hold.

(1) For every subgroup S of G , $f(S)$ is a subgroup of H .

(2) If S' is a subgroup of H , then there exists a subgroup S of G such that $f(S) = S'$.

(3) If S and T are subgroups of G , $S \leq T$ is a necessary and sufficient condition that $f(S) \leq f(T)$.

The correspondence f is a (1-1) correspondence which preserves the partial ordering of the set of subgroups of the group G .

Further, a projectivity f is called index-preserving if $[T:S] = [f(T):f(S)]$ for subgroups S of cyclic subgroups T of G ; and f is called strictly index-preserving if $[T:S] = [f(T):f(S)]$ for subgroups S of subgroups T of G .

If G is the direct product of cyclic groups of order p , p a prime number, R. Baer¹ has given necessary and sufficient conditions that a group H be a projection of G . In particular he has shown that if the projectivity of G upon H is index-preserving, then G and H are isomorphic. Thus in a study of the projections of the prime-power abelian group of order p^m and type $(m-1, 1)$, we need consider only the case $m > 2$.

Rottlaender² investigated the case $m = 3$ and found necessary and sufficient conditions for the existence of a strictly index-preserving projectivity of the prime-power abelian group G of order p^3 and type $(2, 1)$ upon a group H .

In this note, Baer's general results are used to find the necessary and sufficient conditions for the existence of a projectivity of the prime-power abelian group G of order p^m and type $(m-1, 1)$ upon a group H .

2. **The necessary conditions.** If G is an abelian group of the type under consideration, $G = \{u_1\} \times \{u_2\}$ where u_1 is of order p^{m-1} , $m > 2$,

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¹ R. Baer, *The significance of the system of subgroups for the structure of the group*, American Journal of Mathematics, vol. 61 (1939), pp. 1-44. Hereafter this paper will be referred to as B.

² Ada Rottlaender, *Nachweis der Existenz nicht-isomorpher Gruppen von gleicher Situation der Untergruppen*, Mathematische Zeitschrift, vol. 28 (1928), pp. 641-653.