

## SOME REMARKS ON RELATIVELY COMPLETE FIELDS

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In this note we discuss some properties of fields which are relatively complete with respect to valuations of rank one. We show that relatively complete fields have in many respects simpler properties than complete fields. This note arose as a consequence of a discussion of an erroneous statement in a note of one of the authors.<sup>1</sup>

Let  $K$  be a field on which a valuation  $V$  of rank one is defined. Denote the ring of integers in  $K$  with respect to  $V$  by  $O$  and let  $P$  be the prime ideal of nonunits in  $O$ .

DEFINITION<sup>2</sup> 1. *The field  $K$  is termed relatively complete with respect to  $V$  if every polynomial congruence*

$$f(x) \equiv g_0(x)h_0(x) \pmod{P}$$

with  $f(x)$  in  $O[x]$  and  $(g_0(x), h_0(x)) \equiv 1 \pmod{P}$  implies

$$f(x) = g(x)h(x)$$

where  $g(x) \equiv g_0(x) \pmod{P}$  and  $h(x) \equiv h_0(x) \pmod{P}$ .

We remark<sup>3</sup> that there exist relatively complete fields which are not complete with respect to the congruence topology induced by  $V$ . To find examples for such fields it suffices to consider infinite algebraic extensions of a field which is complete with respect to a discrete valuation of rank one.

DEFINITION<sup>4</sup> 2. *Two polynomials  $a(x)$  and  $b(x)$  over a field  $K$  are said to have the same decomposition type if  $a(x) = \prod_{i=1}^s p_i(x)^{m_i}$ ,  $b(x) = \prod_{i=1}^r q_i(x)^{n_i}$  when the  $p_i(x)$  and  $q_i(x)$  are irreducible polynomials and  $s = r$ ,  $m_i = n_i$  and  $p_i(x)$  and  $q_i(x)$  have the same degree for a suitable arrangement of the prime factors.*

We are now able to generalize certain of the results of F. K.

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Presented to the Society, December 31, 1941; received by the editors January 9, 1942.

<sup>1</sup> O. F. G. Schilling, *Remarks on a special class of algebras*, American Journal of Mathematics, vol. 62 (1940), pp. 346–352.

<sup>2</sup> A. Ostrowski, *Untersuchungen zur arithmetischen Theorie der Körper*, Mathematische Zeitschrift, vol. 39 (1935), pp. 269–404.

<sup>3</sup> A. Ostrowski, *Über einige Fragen der allgemeinen Körpertheorie*, Journal für die reine und angewandte Mathematik, vol. 143 (1913), pp. 255–284.

<sup>4</sup> F. K. Schmidt, *Mehrfach perfekte Körper*, Mathematische Annalen, vol. 108 (1933), pp. 1–25.