# TABLE OF THE ZEROS OF THE LEGENDRE POLYNOMIALS OF ORDER 1–16 AND THE WEIGHT COEFFICIENTS FOR GAUSS' MECHANICAL QUADRATURE FORMULA[1]

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Gauss' method of mechanical quadrature has the advantage over most methods of numerical integration in that it requires about half the number of ordinate computations. This is desirable when such computations are very laborious, or when the observations necessary to determine the average value of a continuously varying physical quantity are very costly. Gauss' classical result[2] states that, for the range $(-1, +1)$, the "best" accuracy with $n$ ordinates is obtained by choosing the corresponding abscissae at the zeros $x_1, \cdots, x_n$ of the Legendre polynomials $P_n(x)$. With each $x_i$ is associated a constant $a_i$ such that

$$\int_{-1}^{1} f(x)dx \sim a_1 f(x_1) + a_2 f(x_2) + \cdots + a_n f(x_n). \tag{1}$$

The accompanying table computed by the Mathematical Tables Project gives the roots $x_i$ for each $P_n(x)$ up to $n=16$, and the corresponding weight coefficients $a_i$, to 15 decimal places.

The first such table, computed by Gauss gave 16 places up to $n=7$.[3] More recently work was done by Nyström,[4] who gave 7 decimals up to $n=10$, but for the interval $(-1/2, +1/2)$. B. de F. Bayly has given the roots and coefficients of $P_{12}(x)$ to 13 places.[5]

The Gaussian quadrature formula for evaluating an integral with arbitrary limits $(p, q)$ is given by

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[1] The results reported here were obtained in the course of the work done by the Mathematical Tables Project conducted by the Work Projects Administration for New York City under the sponsorship of the National Bureau of Standards, Dr. Lyman J. Briggs, Director.

[2] *Methodus nova integralium valores per approximationen inveniendi*, Commentationes Societatis Regiae Scientiarum Gottingensis Recentiores, vol. 3 (1814), or *Werke*, vol. 3, pp. 193–195.

[3] It may be found reproduced in Heine's *Kugelfunctionen*, vol. 2, 1881, p. 15, or Hobson, *Spherical Harmonics*, pp. 80–81.

[4] Nyström, Acta Mathematica, vol. 54 (1930), p. 191.

[5] B. de F. Bayly, Biometrika, vol. 30 (1938), pp. 193–194.