

where $s_v^*(\theta_0)$, $v = 0, 1, \dots, n$, is the sequence $|s_v(\theta_0) - s|$ in decreasing order.

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2. ———, *Notes on the theory of series (XVIII): On the convergence of Fourier series*, Proceedings of the Cambridge Philosophical Society, vol. 31 (1935), pp. 317-323.
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THE BASIC ANALOGUE OF KUMMER'S THEOREM¹

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1. **Introduction.** About one hundred years ago, E. E. Kummer² proved the formula

$$(1) \quad {}_2F_1 \left[\begin{matrix} a, & b; & -1 \\ 1+a-b \end{matrix} \right] = \frac{\Gamma(1+a-b)\Gamma(1+a/2)}{\Gamma(1+a)\Gamma(1+a/2-b)}$$

which has since been known as Kummer's theorem. This appears to be the simplest relation involving a hypergeometric function with argument (-1) .

All the relations in the theory of hypergeometric series ${}_rF_s$, which have analogues in the theory of basic series³ are those in which the argument is $(+1)$. Apparently, there has been no successful attempt to establish the basic analogue of any formula involving a function ${}_rF_s(-1)$. Since Kummer's theorem is fundamental in the proofs of numerous relations between hypergeometric functions of argument (-1) , it seemed desirable that an attempt be made to prove the basic analogue of Kummer's theorem and to investigate the possibility of obtaining new relations in basic series with arguments corresponding to the argument (-1) in the classical case.

In this paper, the basic analogue of Kummer's theorem is obtained

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¹ The results presented in this paper are included in a dissertation for the doctorate, University of Nebraska, 1941.

² E. E. Kummer, *Ueber die hypergeometrische Reihe*, Journal für die reine und angewandte Mathematik, vol. 15 (1836), pp. 39-83.

³ W. N. Bailey, *Generalized Hypergeometric Series*, Cambridge Tract, no. 32, chap. 8.