ON THE LOGARITHMIC MEANS OF REARRANGED PARTIAL SUMS OF A FOURIER SERIES

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Let $f(\theta)$ be a real, even and Lebesgue integrable function; let

$$f(\theta) \sim (1/2)a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta.$$

We write

$$s_0 = (1/2)a_0, \qquad s_n = (1/2)a_0 + a_1 + \cdots + a_n, \qquad n \geqq 1,$$

and denote by $s_0^*, s_1^*, \cdots, s_n^*$ the values of $|s_0|, |s_1|, \cdots, |s_n|$ rearranged in decreasing order. In 1935 Hardy and Littlewood [2][1] proved the following remarkable theorem:

THEOREM 1. *If*

$$f(\theta) = o\left(\log \frac{1}{\theta}\right)^{-1} \tag{1}$$

for small positive θ, *then*

$$\sum_{0}^{n} \frac{s_v^*}{v+1} = o(\log n). \tag{2}$$

Hardy and Littlewood gave two applications of this theorem by proving:

THEOREM 2. *If* (1) *holds, then*

$$\sum_{1}^{n} |s_v|^q = o(n) \tag{3}$$

for every positive q.

THEOREM 3. *If* (1) *holds and if*

$$a_n > -An^{-\xi} \tag{4}$$

for a positive A *and* ξ, *then* $s_n \to 0$.

They have also proved [1, Theorem 9] that in Theorem 3 the

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[1] Numbers in brackets refer to the bibliography at the end of this paper.