ANALYSIS

255. C. R. Adams and A. P. Morse: *On approximating certain integrals by sums.*

For $f \in L(E)$, B a measurable subset of E, $0<|B|=$measure $(B)<\infty$, let $\mathfrak{M}_B f=\int_B f/|B|$. As B varies, let $\mathfrak{R}(f)$ represent the set of values of $\mathfrak{M}_B f$; and let ϕ be a function whose domain includes $\mathfrak{R}(f)$. For $0<\delta\leqq\infty$ let F be an arbitrary set-partition of E into disjoint measurable subsets each with diameter less than δ; and let the aggregate of all such partitions be denoted by $\Gamma_\delta(E)$. What conditions on f and ϕ will insure the (finite) existence of $\int_E \phi[f(x)]dx$ and of $\lim_{\delta\to 0} \inf_{F\in\Gamma_\delta(E)} \sum_{B\in F} \phi[\mathfrak{M}_B f]|B|$, $\lim_{\delta\to 0} \sup_{F\in\Gamma_\delta(E)} \sum_{B\in F} \phi[M_B f]|B|$ and their equality? For ϕ continuous, a necessary and sufficient condition is found. The hypothesis of continuity on ϕ cannot be dispensed with. "Sampling" can be allowed in the sum (see Adams and Morse, *Random sampling in the evaluation of a Lebesgue integral*, this Bulletin, vol. 45 (1939), pp. 442–447). A sufficient condition, often useful for testing, is found in terms of the existence of a convex dominant for $|\phi|$; such a convex dominant need not exist, but a condition is determined under which it does. Applications are made to functions f which are of bounded variation or are absolutely continuous in a certain generalized sense involving ϕ. Some new results in the general theory of functions of sets are included. (Received July 14, 1942.)

256. G. E. Albert: *Criteria for the closure of systems of orthogonal functions.*

Let the system F of functions $f_n(x)$, $n=0, 1, 2, \cdots$, be orthonormal on the interval (a, b). For any fixed point t in (a, b) let $g_t(x)$ denote the function which is equal to unity on (a, t) and zero on (t, b). Let $s_n(x)$ denote the partial sum of the generalized Fourier series with respect to F for the function $g_t(x)$. Define the function $\sigma_n(t)$ which, for each t in (a, b), is equal to $s_n(t)$. A necessary and sufficient condition that the system F be closed in the class of functions having integrable (Riemann or Lebesgue) squares on (a, b) is: $\lim_n \int_a^b |1-2\sigma_n(t)|dt=0$. A sufficient condition is that $\lim_n \int_a^b \{1-2\sigma_n(t)\}^2 dt=0$. The verification of the latter criterion for the trigonometric system F is a matter of elementary calculus. Both criteria are extended to systems F orthogonal with respect to a positive weight function; in such cases the interval (a, b) may be infinite. The criteria stated follow easily from a theorem due to Vitali (Rendiconti dei Lincei, (5), vol. 30 (1921)). (Received June 6, 1942.)

257. R. H. Cameron and W. T. Martin: *Infinite linear difference equations with arbitrary real spans and first degree coefficients.*

The authors investigate the equation $\int_{-\infty}^{\infty}(z-\lambda)f(z-\lambda)dp(\lambda)+\int_{-\infty}^{\infty}f(z-\lambda)dq(\lambda)=g(z)$ in a strip $a<lmz<b$. Under fairly weak conditions on p, q, and g it is shown that the equation has a unique analytic solution of a fairly general character. (Received June 24, 1942.)

258. J. A. Clarkson and Paul Erdös: *On the approximation of continuous functions by polynomials.*

Let x^{n_i} be a set of powers of x, $n_i\to\infty$. Then a well known theorem of Müntz and Szász states that the necessary and sufficient condition that the powers x^{n_i} and 1 shall span the whole space of continuous functions, in the interval (0, 1) is that