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PURDUE UNIVERSITY

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## INVERSES AND ZERO-DIVISORS

REINHOLD BAER

It may happen that an element in a ring is both a zero-divisor and an inverse, that it possesses a right-inverse though no left-inverse, and that it is neither a zero-divisor nor an inverse. Thus there arises the problem of finding conditions assuring the absence of these paradoxical phenomena; and it is the object of the present note to show that chain conditions on the ideals serve this purpose. At the same time we obtain criteria for the existence of unit-elements.

The following notations shall be used throughout. The element  $e$  in the ring  $R$  is a *left-unit for the element  $u$*  in  $R$ , if  $eu = u$ ; and  $e$  is a *left-unit for  $R$* , if it is a left-unit for every element in  $R$ . Right-units are defined in a like manner; and an element is a *universal unit for  $R$* , if it is both a right- and a left-unit for  $R$ .

The element  $u$  is a *right-zero-divisor*, if there exists an element  $v \neq 0$  in  $R$  such that  $vu = 0$ ; and  $u$  is a *right-inverse in  $R$* , if there exists an element  $w$  in  $R$  such that  $wu$  is a left-unit for  $u$  and a right-unit for  $R$ . Left-zero-divisors and left-inverses are defined in a like manner. Note that 0 is a zero-divisor, since we assume that the ring  $R$  is different from 0.

$L(u)$  denotes the set of all the elements  $x$  in  $R$  which satisfy  $xu = 0$ ; clearly  $L(u)$  is a left-ideal in the ring  $R$  and every left-ideal of the form  $L(u)$  shall be termed a *zero-dividing left-ideal*. *Principal left-ideals*<sup>1</sup> are the ideals of the form  $Rv$  for  $v$  in  $R$  and the ideals  $vR$  are the principal right-ideals.

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<sup>1</sup> This is a slight change from the customary terminology.