A NEW PROOF OF THE CYCLIC CONNECTIVITY THEOREM

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The cyclic connectivity theorem was first proved for the plane in 1927 by G. T. Whyburn [5]. The extension of this theorem to metric space afforded some difficulty and the first proof [1] was long and tedious and complicated with convergence difficulties. A second and simpler proof appeared in 1931 [6], but in this proof it is necessary that quite a number of properties of Peano spaces be proved in advance.

This note attempts to give a new proof in which convergence troubles are encountered at just one point (step (b)) and in which just three theorems about Peano space need be known in advance: (A) *Every component of an open set is open.* (B) *Open connected sets are arc-wise connected.* (C) *The space is arc-wise locally connected.* Actually just two properties need to be established before cyclic connectivity can be proved, for the third theorem (C) is a simple consequence of the first two.[1] Thus the cyclic connectivity theorem may be established at the very beginning of the theory of Peano spaces and is available for use in studying other properties.

CYCLIC CONNECTIVITY THEOREM. *If no single point of a locally compact, connected and locally connected metric space separates the space between the two given points, there is a simple closed curve containing the two points.*

Let p and q be the two points. There exists an arc α of the space S with end points p and q by (B). We shall say that an arc β *spans* the point v of α if β has only its end points on α and v lies between these end points. We shall say that a set of arcs C *spans* a subset K of α if each point of K is spanned by some arc of the set C.

If an arc β exists with end points r and q and such that $\alpha\cdot\beta=r+q$, then step (d) in the proof has been achieved. Hence we consider only the case where no such arc exists. This assumption is used in the proof of step (b).

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[1] Let G be the component of $S(p, \epsilon)$ containing the point p. By (A), G is open. Then for some δ, $S(p, \delta)\subset G$. By (B) G is arc-wise connected. Hence every point of $S(p, \delta)$ may be joined to p by an arc in G, and thus in $S(p, \epsilon)$, which proves arc-wise local connectivity.