GENERALIZED FISCHER GROUPS AND ALGEBRAS[1]

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Introduction. In this paper we shall be concerned with the structure of the rational representation of certain sets of matrices, to which we give the name *generalized Fischer sets.* If K is any field, ϕ any fixed automorphism of K, and A any matrix with elements in K, we use the notation A^ϕ for the ϕ-automorph of A; that is, the matrix obtained from A by subjecting each of its elements to the automorphism ϕ. Again, we denote by A^* the transposed ϕ-automorph A'^ϕ of A.

DEFINITION. *A set $\mathfrak{M}$ of n-rowed square matrices which contains A^* if it contains A is defined to be a generalized Fischer set.*

Generalized Fischer groups, semi-groups and algebras are similarly defined.

In either of the special cases (1) $K=k$, a real field, ϕ is the identity automorphism; (2) $K=k^+=k(-1)^{1/2}$, ϕ is the operation of taking the conjugate complex, the set $\mathfrak{M}$ will be called a *Fischer set.* Fischer groups were probably named[2] by M. Schiffer, who, in 1933, proved in an unpublished work that every such group is completely reducible. This result has also been given by Specht [**3**], and will again be derived for all Kronecker product representations in the present paper (Theorem I, §4). In §1 we give a partial converse in the cases of the field of all reals (Example (6)), and the field of all complex numbers (Example (5)); this is summed up in Theorem II (§4).

Unlike Fischer sets, generalized Fischer sets and their rational representations are not always completely reducible; the regular representation of a finite group over a field of prime characteristic dividing the order of the group is a case in point (§1, Example (8)). When ϕ is non-involutary, the most we can give concerning the structure of g.F. sets is contained in Lemma II (§4) and Lemma IV (§5). But when ϕ is an involutary automorphism, a more satisfactory result is

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[1] The following is essentially contained in the author's doctorate thesis [**1**], written under the direction of Professor Richard Brauer. Professor Brauer has also offered many helpful suggestions in connection with the present paper. The thesis undertook a general study of $GL(n)$, and employed the results for specific calculation of the irreducible characters of $GL(4)$ over an infinite modular field.

[2] They were considered earlier by E. Fischer [**2**], who proved that the rational integral invariants of a Fischer group possessed a finite integrity basis.