*be the orthonormal trigonometric sums on $D_1$ for weight $\rho(\theta)\sigma(\theta)$; if the u's and v's are uniformly bounded on a point set $D_2$ contained in $D_1$, the same is true of the U's and V's.*

For this case the proof admits a materially simpler formulation than when geometric configurations are contemplated having the degree of generality previously considered. The details relating to the loci $C'$, $C''$, $K$, $K'$, $K''$, can be dispensed with for the most part; with $\theta$ replacing the pair of coordinates $(x, y)$, and $\phi$ replacing the pair $(u, v)$, it is sufficient, for any particular value of $\theta$, to consider separately the intervals $(\theta-\pi/2, \theta+\pi/2)$ and $(\theta+\pi/2, \theta+3\pi/2)$, and in the integral corresponding to the right-hand member of (5) to represent $K_{n-1}(\theta, \phi)$ in the former interval by an expression with denominator $\sin(\theta-\phi)$, and in the latter interval by an alternative expression with $1-\cos(\theta-\phi)$ in the denominator.[10]

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[10] [B, pp. 808–809.]

# APPROXIMATION OF CONTINUOUS FUNCTIONS BY MEANS OF LACUNARY POLYNOMIALS

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**1. Introduction.** All rational integral polynomials are linear combinations of members of the complete set of powers whose exponents are the non-negative integers. If certain members of this set are deleted, the linear combinations formed from the resulting set are, in the strict sense of the term, "lacunary polynomials." In a large part of this paper, however, methods of reasoning designed for the treatment of such polynomials are applicable to combinations from much more general sets of powers whose exponents are non-negative but not in general integral. The term "polynomial in $x^\mu$ of degree $\mu_n$" will be applied to combinations from the set $1, x^{\mu_1}, x^{\mu_2}, \cdots$ where $\mu_1, \mu_2, \cdots$ form an arbitrarily preassigned set of real numbers such that $0<\mu_1<\mu_2<\cdots$, and $\mu_n$ is the largest exponent.

This paper started out as an investigation of lacunary orthogonal polynomials, and although this aspect of it became subordinate to the

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