GENERALIZATION OF A THEOREM OF KOROUS ON THE BOUNDS OF ORTHONORMAL POLYNOMIALS

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1. **Introduction.** An elementary treatment of the convergence of series of orthogonal polynomials is greatly facilitated if the polynomials of the orthonormal set are known to be uniformly bounded on the domain of orthogonality, or on a part of it where convergence is to be proved.[1] A demonstration due to J. Korous[2] shows in a few lines that the orthonormal polynomials corresponding to a weight function $\rho\sigma$ on a finite interval are thus bounded, if the polynomials for weight ρ have the desired property, and if the factor σ satisfies a Lipschitz condition and has a positive lower bound on the entire domain of orthogonality. The purpose of this note is to show that the argument of Korous can be extended so as to apply under fairly general conditions to orthogonal polynomials in two real variables on an algebraic curve[3] and in particular to orthogonal trigonometric sums,[4] which can be regarded as orthogonal polynomials on a circle.[5] A problem of the same category has been discussed by Peebles[6] with less simple hypotheses on the factor σ.

In the case of trigonometric sums it is known in advance that the orthonormal functions for weight $\rho\equiv 1$, namely $(2\pi)^{-1/2}$, $\pi^{-1/2}\cos kx$, $\pi^{-1/2}\sin kx$, $k=1, 2, \cdots$, are uniformly bounded, and a theory of the convergence of developments in series of orthogonal trigonometric sums is opened up immediately. For other algebraic curves the question of the existence of a weight function which gives rise to a bounded system of orthonormal polynomials is one requiring separate investigation, and the answer to this question is known at present only in particular instances.[7] When the existence of a single such

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[1] See, for example, D. Jackson, *Series of orthogonal polynomials*, Annals of Mathematics, (2), vol. 34 (1933), pp. 527–545; pp. 531–538.

[2] See G. Szegö, *Orthogonal Polynomials*, American Mathematical Society Colloquium Publications, vol. 23, 1939, p. 157; D. Jackson, *Fourier Series and Orthogonal Polynomials*, Carus Mathematical Monographs, no. 6, 1941, pp. 205–208.

[3] See D. Jackson, *Orthogonal polynomials on a plane curve*, Duke Mathematical Journal, vol. 3 (1937), pp. 228–236. This paper will be cited by the letter A.

[4] See, for example, D. Jackson, *Orthogonal trigonometric sums*, Annals of Mathematics, (2), vol. 34 (1933), pp. 799–814. This paper will be cited as B.

[5] [A, p. 234.]

[6] G. H. Peebles, this Bulletin, abstract 45-5-219.

[7] See, for example, D. Jackson, this Bulletin, abstract 45-5-192; Fulton Koehler,