uniquely as a product of a finite number of prime differential ideals. The consistency of the axioms is easily shown. If we define differentiation in the ring $C(x)$, obtained by adjoining x to the field of the rational numbers, in *any* way so as to leave it closed, it may be shown that Axioms I–IV are always satisfied. In $C(x, y)$ differentiation *may* be defined in such way that the statement as above still holds. This is of interest because every ordinary ideal in $C(x, y)$ may not be expressed uniquely as a product of a finite number of prime ideals.

HOFSTRA COLLEGE

ON THE ITERATION OF LINEAR HOMOGENEOUS TRANSFORMATIONS

ARNOLD DRESDEN

1. **Statement of problem.** The question which this note tries to answer is that of determining under what conditions on the matrix (a_{ij}), $(i, j=1, \cdots, n)$, the n-fold multiple sequence of complex numbers $x_k, x_k', \cdots, x_k^{(m)}, \cdots$ $(k=1, 2, \cdots, n)$ obtained by iteration of the linear homogeneous transformation $x_k' = a_{kj}x_j$ will converge for every initial set $x=(x_1, x_2, \cdots, x_n)$. Convergence is to be understood in the sense that there exists a set $X_1, X_2, \cdots, X_n$ such that, for $k=1, 2, \cdots, n$, $x_k^{(m)} \to X_k$, as $m \to \infty$.

2. **Jordan normal form.** We begin by recalling that a matrix $A=(a_{ij})$ with complex elements is similar to its Jordan normal form J_0. This means that there exists a unimodular matrix P, such that $A=P^{-1}J_0P$ and $J_0=PAP^{-1}$, where J_0 is the direct sum of Jordan matrices $J_1, \cdots, J_N$. To each elementary divisor $(\lambda-\lambda_\rho)^{e_\rho}$ of the characteristic matrix $\lambda I-A$ $(\rho=1, 2, \cdots, N)$ and $e_1+e_2+\cdots+e_N=n$, corresponds a Jordan matrix J_ρ. If $e_\rho>1$, then J_ρ has zero elements everywhere, except in the principal diagonal, all of whose elements are λ_ρ, and in the diagonal immediately below the principal diagonal, all of whose elements are 1.[1] If $e_\rho=1$, then J_ρ consists of the single element λ_ρ.

It follows that any integral power of J_0 is the direct sum of the same powers of the Jordan matrices J_ρ. Let us now denote by J an arbitrary Jordan matrix of order $n>1$,

---

Received by the editors August 25, 1941.

[1] See, for example, MacDuffee, *Introduction to Abstract Algebra*, p. 241.