FACTORIZATION OF DIFFERENTIAL IDEALS

HENRY J. RIBLET

1. **Introduction.** With the ultimate objective of introducing number theoretical considerations into the theory of algebraic differential fields,[1] conditions on a differential ring $\mathfrak{R}$ are given which are sufficient to insure that any differential ideal in it may be uniquely expressed as the product of a finite number of prime differential ideals. This program is justified by its intrinsic interest and the fact that the existence of such conditions suggests the possibility of obtaining a number theoretical complement for the theory of differential ideals in the ring of differential forms as developed by Ritt, Raudenbush and others,[2] paralleling the interdependent, classical theories of polynomial and number ideals.

With an additional axiom and certain alterations of the definitions, the proof of the basic theorems follows the pattern used by van der Waerden[3] in presenting the method of Krull.[4] Our procedure will be to discuss in detail only those questions raised by the altering of definitions.

2. **Definitions.** By a *differential ring* $\mathfrak{R}$, we shall mean a commutative ring with a unit element but no divisors of zero which is closed under differentiation. Its quotient field will be denoted by $\mathfrak{K}$. A *differential ideal* $\mathfrak{a}$ in $\mathfrak{R}$ is an ordinary ideal closed with respect to differentiation. $\mathfrak{R}$ will satisfy the *ascending chain condition* for differential ideals if every sequence of differential ideals, each properly containing the previous one, is finite in length. A differential ideal is said to be *divisorless* if the only differential ideal which properly contains it is the unit ideal $\mathfrak{R}$. A *principal* differential ideal is a differential ideal which is principal in the usual sense. The unit ideal is an example. $\mathfrak{R}$ will be said to be *integrally closed* if each element in $\mathfrak{K}$, having the property that all its integral powers have a fixed denominator, is in $\mathfrak{R}$. A differential ideal $\mathfrak{p}$ in $\mathfrak{R}$ will be called

Presented to the Society, April 26, 1940 under the title *Factorization of differential ideals in an algebraic field*; received by the editors November 6, 1941.

[1] H. J. Riblet, *Algebraic differential fields*, American Journal of Mathematics, vol. 53 (1941), pp. 339–446.

[2] J. F. Ritt, *Algebraic aspects of the theory of differential equations*, American Mathematical Society Semicentennial Publications, vol. 2, bibliography.

[3] B. L. van der Waerden, *Moderne Algebra*, vol. 2, p. 85.

[4] W. Krull, *Zur Theorie der allgemeinen Zahlringe*, Mathematische Annalen, vol. 99 (1928), pp. 51–70.