SOME NEW METHODS OF SOLUTION OF TWO-DIMENSIONAL PROBLEMS IN ELASTICITY

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1. **Introduction.** The main purpose of this address is to bring to the attention of the workers in the theory of elasticity and related branches of applied mathematics a simple general method of solution of several important classes of two-dimensional boundary value problems. I use the term "two-dimensional" or "plane" boundary value problems in the sense that their mathematical formulation requires the introduction of only two independent variables. In this sense the problems of St. Venant on torsion and flexure of cylinders, and the problems on deflection and buckling of elastic plates, which have a three-dimensional physical aspect, are two-dimensional.

The method which I intend to discuss was developed mainly by a group of Russian mathematicians, and despite the fact that it has been utilized extensively in Russia for more than a decade, it is virtually unknown in this country. A great variety of problems to which it has been applied to obtain useful solutions includes an investigation of flexure and torsion of beams, a study of thermo-elastic stresses in composite cylinders, an analysis of deflection of anisotropic plates, and a multitude of problems characterized by the states of plane stress and plane strain.

Inasmuch as familiarity with the concepts of applied mathematics is a rare virtue, I shall reduce the use of the technical language to a minimum, and shall ask you to take for granted certain basic equations of the theory of elasticity. Failure to comprehend the origin of these equations will not impair the understanding of the general method of their solution.

We shall suppose that a two-dimensional region R, occupied by an elastic medium, is referred to a system of cartesian axes (x, y). To fix the ideas we can think of the region R as representing the cross-section of a long cylinder whose elements are parallel to the z-axis, and whose lateral surface is subjected to a distribution of external forces that is independent of the z-coordinate. Under the action of such forces, the medium, in general, will be distorted, and the displacement of the points of the region R in the directions of the x- and y-axes will be denoted by $u(x, y)$ and $v(x, y)$, respectively. If the medium

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