233. H. Schwerdtfeger: *A complete parametrization of the symplectic group.*

While all known parametrizations of the symplectic group omit certain "exceptional" elements, the parametrization derived in the present paper covers the whole group. Let ϵ be the n-rowed unit matrix and F the matrix with first row 0,0 and second row ϵ,0. A $2n$-rowed real regular matrix T is proved to be symplectic if and only if $T'FT-F=H$ is symmetric (contact condition), and satisfies the condition: $(F'-F)\cdot(H+F)$ is idempotent. To prove the sufficiency of the latter condition one has to show that for any such matrix H a symplectic T can be found with which it is associated by the said relation. By establishing a set of normal forms for H under the transformation $S'HS$ where S is a $2n$-rowed matrix for which $S'FS=F$, that is, S is the matrix with first row σ,0 and second row 0,σ'^{-1}, a set of normal matrices T can be found such that RTS is the most general symplectic matrix where R is the matrix with first row ρ,0; second row 0,ρ'^{-1} and ρ,σ are any regular n-rowed matrices. The method has been carried through in detail for $n=2$. (Received May 1, 1942.)

Analysis

234. Einar Hille: *Notes on linear transformations.* IV. *Representation of semi-groups.*

Let $\{T_s\}$ be a semi-group of linear bounded transformations on a separable Banach space to itself, defined for $s>0$. Let T_s be weakly measurable and $\|T_s\|\leqq 1$ for $s>0$. Let $T_s\ (E)$ be dense in E. Put $A_h=(1/h)[T_h-I]$. For $h\to 0$, $T_hx\to x$ everywhere in E and $A_hx\to Ax$ in a dense set $D(A)$. Here A is linear, closed and ordinarily unbounded. The resolvent $R(\lambda)$ of A is the negative of the Laplace transform of T_s and is bounded for $\Re(\lambda)>0$. Conversely, T_s is expressible in terms of $R(\lambda)$ by the inversion formula for Laplace integrals which gives an interpretation of T_s as $\exp(sA)$. A further interpretation is given by $T_sx=\lim_{h\to 0}\exp[sA_h]x$, valid in $D(A)$. The method is essentially that of Stone. (Received April 6, 1942.)

235. Witold Hurewicz: *An ergodic theorem without invariant measure.*

Let E be an abstract space carrying a completely additive measure μ defined on a completely additive field Ω of subsets of E (it is assumed that a set $X\in\Omega$ with $\mu(X)=\infty$ can always be split into a countable number of sets with finite measures). Let T be a one-to-one point transformation of E on itself satisfying the conditions: (1) $X\in\Omega$ implies $T(X)\in\Omega$; (2) $T(X)\subset X\in\Omega$ implies $\mu(X-T(X))=0$ (incompressibility condition). Finally let $F(X)$ $(X\in\Omega)$ be an additive finite-valued set function, absolutely continuous with respect to the measure μ. For $X\in\Omega$, let $F_n(X)=\sum_{i=0}^{n}F(T^i(X))$, $\mu_n(X)=\sum_{i=0}^{n}\mu(T^i(X))(n=0, 1, 2, \cdots)$. Since F_n is absolutely continuous with respect to the measure μ_n, $F_n(X)=\int_X f_n d\mu_n$, where f_n is a measurable point function defined on E. It can be shown that the sequence $\{f_n\}$ converges almost everywhere to a function ϕ, invariant with respect to T. By "almost everywhere" is meant that the points of divergence form a set M such that $\mu(T^n(M))=0$ for $n=1, 2, \cdots$. If the measure μ is finite and invariant with respect to T, this theorem coincides with Birkhoff's ergodic theorem. (Received April 24, 1942.)

236. William Karush: *A sufficiency theorem for an isoperimetric problem in parametric form with general end conditions.*

The problem studied is that of minimizing an integral $I(C)=g(a)+\int_{t_1}^{t_2}f(a, y, y')dt$