ON THE MAPPING OF n QUADRATIC FORMS

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If $Q_1(z), Q_2(z), \cdots, Q_n(z)$ are quadratic forms in the real variables $z^1, z^2, \cdots, z^m$ with real coefficients, the question arises as to the conditions under which there exists a linear combination

$$\lambda_1 Q_1(z) + \lambda_2 Q_2(z) + \cdots + \lambda_n Q_n(z)$$

which is positive definite. A number of recent papers have considered this question.

For the case $n=2$ a satisfactory answer was obtained by Paul Finsler,[1] and independently by a group of interested persons at the University of Chicago.[2]

For any finite n, Finsler obtained sufficient conditions under the restriction that the number of independent variables does not exceed four. The conditions are quite involved, and are in terms of a certain type of algebraic manifold which Finsler designates "Freigebilde" and of which he had made an elaborate study in an earlier paper.

For any finite n and any finite number of independent variables Hestenes and McShane, in a joint paper,[3] obtained sufficient conditions. They are obviously not necessary, and though they seem exactly suited to the application which the authors desired to make, their lack of symmetry perhaps leaves something to be desired.

In a recent paper[4] the present author called attention to the suitability of the theory of convexity as a means for studying this type of question, and treated the case of two quadratic forms in m variables from this point of view. The purpose of the present paper is to make a similar study of the general case of n quadratic forms in m variables.

1. **The map $\mathfrak{M}$.** The n quadratic forms $Q_1(z), Q_2(z), \cdots, Q_n(z)$ may be thought of as mapping the m-dimensional space $\mathfrak{Z}_m$ of points $z=(z^1, z^2, \cdots, z^m)$ onto an n-dimensional space $\mathfrak{X}_n$ of points

Presented to the Society, December 31, 1941; received by the editors September 5, 1941.

[1] *Über das Vorkommen definiter und semidefiniter Formen in Scharen quadratischer Formen*, Commentarii Mathematici Helvetici, vol. 9 (1937), pp. 188–192.

[2] From this group two different proofs of the essential theorem appeared: one by A. A. Albert, this Bulletin, vol. 44 (1938), pp. 250–253; and one by W. T. Reid, ibid., pp. 437–440.

[3] *A theorem on quadratic forms and its application in the calculus of variations*, Transactions of this Society, vol. 47 (1940), pp. 501–512.

[4] *On the mapping of quadratic forms*, this Bulletin, vol. 47 (1941), pp. 494–498.