ON 3-DIMENSIONAL MANIFOLDS

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Let P be a 3-dimensional manifold.[1] Let Q be a 2-dimensional manifold imbedded in P. Moreover, let P and Q admit of a *permissible simplicial division* K, that is, a simplicial division of P such that some subcomplex of K, say L, is a simplicial division of Q. Let K_i and L_i denote the ith normal subdivisions of K and L, respectively. We define the *neighborhood* N_i of L_i to be the simplicial complex consisting of the simplexes of K_i that have at least one vertex in L_i together with the sides of all such simplexes. By the *boundary* B_i of N_i we mean the simplicial complex consisting of the simplexes of N_i that have no vertex in L_i. Our purpose is to prove the following theorem.

THEOREM. *The boundary B_2 is a two-fold but not necessarily connected covering of Q, and change of permissible division K replaces B_2 by a homeomorph of itself.*

PROOF. The neighborhood N_1 is the sum of a set of 3-dimensional simplexes. Some of these 3-simplexes, say $a_1, a_2, \cdots$, have exactly one vertex in L_1, others, say $b_1, b_2, \cdots$, have exactly two vertices in L_1, while the remaining, say $c_1, c_2, \cdots$, have three vertices in L_1. Since K_1 is a normal subdivision of K, the intersection of L_1 and b_i or c_i is a 1-simplex or 2-simplex, respectively. Let α_i, β_i, and γ_i be the intersections of B_2 and a_i, b_i, and c_i, respectively. We shall regard α_i and γ_i as triangles with vertices on the 1-simplexes of a_i and c_i. Also we shall regard β_i as a square with vertices on the 1-simplexes of b_i.

Any 2-simplex of L_1, say ABC, is incident to exactly two of the c_i. Let $c_1=ABCM$. There is a unique 3-simplex of N_1, say σ, that is incident to ABM and different from c_1. This σ is either a c_i, say c_2, or a b_i, say b_2. If σ is c_2, then the triangles γ_1 and γ_2 have a common side. Suppose that σ is $b_2=ABMN$. The 2-simplex ABN is incident to a unique 3-simplex of N_1, say τ, with $\tau\neq ABMN$. This τ is either c_3 or b_3. If $\tau=b_3$, there is a c_4, or b_4. Finally we must find a $c_p=ABDS$, D in L_1, S in B_1. We now consider $\beta_2, \beta_3, \cdots$, and β_{p-1}. The sum of these squares is topologically equivalent to a square. One side of the square is coincident with a side of γ_1 and the opposite side coincident with a side of γ_p.

Received by the editors July 21, 1941.

[1] Our terminology is that of Seifert-Threlfall, *Lehrbuch der Topologie*. Manifolds are finite, while simplexes and cells are closed point sets.