all integral functions $f(z)$ satisfying the conditions $f(t)\in L_1$, $\mathfrak{H}f\in L_1$, $|f(z)| < K_{f,\epsilon} \exp \{(2\alpha+\epsilon)|z|\}$. The proof is based upon a result due to Plancherel and Pólya.[12]

THE UNIVERSITY,
EDGBASTON, BIRMINGHAM, ENGLAND

[12] Commentarii Mathematici Helvetici, vol. 10 (1937–1938), pp. 110–163, §27.

THE BEHAVIOR OF CERTAIN STIELTJES CONTINUED FRACTIONS NEAR THE SINGULAR LINE

H. S. WALL

1. **Introduction.** We consider here continued fractions of the form[1]

$$(1.1)\qquad f(z) = \frac{g_0}{1} + \frac{g_1 z}{1} + \frac{(1-g_1)g_2 z}{1} + \frac{(1-g_2)g_3 z}{1} + \cdots,$$

in which $g_0 \geqq 0$, $0 \leqq g_n \leqq 1$, $(n=1, 2, 3, \cdots)$, it being agreed that the continued fraction shall terminate in case some partial numerator vanishes identically. There exists a monotone non-decreasing function $\phi(u)$, $0 \leqq u \leqq 1$, such that

$$(1.2)\qquad f(z) = \int_0^1 \frac{d\phi(u)}{1+zu};$$

and, conversely, every integral of this form is representable by such a continued fraction. Put $M(f) = \text{l.u.b.}_{|z|<1} |f(z)|$. Then $M(f) \leqq 1$ if and only if the continued fraction can be written in the form

$$(1.3)\qquad f(z) = \frac{h_1}{1} + \frac{(1-h_1)h_2 z}{1} + \frac{(1-h_2)h_3 z}{1} + \cdots,$$

in which $0 \leqq h_n \leqq 1$, $(n=1, 2, 3, \cdots)$. These functions are analytic in the interior of the z-plane cut along the real axis from $z=-1$ to $z=-\infty$.

The principal object of this paper is to prove the following theorem:

THEOREM 1.1. *If* $0 < h_n < 1$, $(n=1, 2, 3, \cdots)$, *and* $h_n \to 1/2$ *in such a way that the series* $\sum |h_n - 1/2|$ *converges, then the function* $f(z)$ *given*

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[1] H. S. Wall, *Continued fractions and totally monotone sequences*, Transactions of this Society, vol. 48 (1940), pp. 165–184.