

TRENDS IN RESEARCH: THE ANALYTIC NUMBER THEORY

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Ladies and Gentlemen: I wish to express my gratitude for the invitation to address this meeting and to thank you for granting me this opportunity to contribute a little part to the celebration of the anniversary of our host, this renowned institution of higher learning, the University of Chicago. The research work emanating from here has in its mathematical branch strongly emphasized analysis as well as number theory. The name "analytic number theory," indicating a union of these two fields of mathematical endeavor, is about as old as this celebrating Institution. In 1894 a book was published under the title *Die Analytische Zahlentheorie*, by Paul Bachmann. The name "analytic number theory" itself was a program. It said more than "diverse applications of infinitesimal calculus to number theory" as Dirichlet, the real founder of our discipline, had modestly called one of his great memoirs (1839). The name "analytic number theory," implies, as I take it, a thorough fusion of analysis and arithmetic, in which as we shall see, analysis is not necessarily subordinate to arithmetic. Incidentally, "analysis" stands here always for function theory, especially the theory of analytic functions.

What has the theory of functions to do with the theory of integers? Let me, to gain a starting point, begin with a few examples of well established results of analytic number theory:

1. If $\pi(x)$ denotes as usual the number of prime numbers less than or equal to x , then

$$(1) \quad \lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x} = 1$$

as first proved by Hadamard and de la Vallée Poussin in the nineties of the last century.

2. The number of representations of a positive integer n as a sum of four squares is

$$(2) \quad r_4(n) = 8 \left(\sum_{d|n} d - \sum_{4|d'|n} d' \right)$$

as Jacobi found as a corollary of his theory of elliptic functions.

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