

ment of this theory is use of the relationship between V_m and the symmetric group of degree m . For k of characteristic p the classical theory is valid only for $m < p$. In the present paper the decomposition of V_m is obtained for $p \leq m < 2p$. Further progress on the decomposition problem will likely have to await the development of the theory of modular representations of the symmetric groups of degree $m \geq 2p$; the measure of difficulty here being the power of p which divides $m!$. (Received March 26, 1942.)

167. Bernard Vinograd: *Split rings and their representation theory.*

Suppose a ring R , with minimum condition on left ideals, can be split into a direct sum, $R = R^* + N$, of a semi-simple ring and the radical. This splitting property of R is equivalent to the existence of a certain set of noncommutative fields in R . Let R have a unit, and let V be a commutative group possessing R in its left operator domain and a finite composition series with respect to R . Then V is a direct sum of modules, each over one of the noncommutative fields. Thus V gives rise to a composite representation module for its homomorphism ring. In the case $V = R$ the resulting representation of R displays the essential structure of R^* and N , and may be considered a natural extension of the Wedderburn theorem for simple rings to the class of split rings with unit and finite composition series. (Received March 20, 1942.)

168. R. W. Wagner: *Expressibility relations for bilinear operations.* Preliminary report.

If xy is a multiplication operation for a vector space over a field, an expression like $(a((xb)(cy)))d$ is also a multiplication for the vector space. The operation x^*y is called simply expressible in the operation xy if x^*y is a finite iteration of the original operation. The operation x^*y is called strongly expressible if it is a linear combination of weakly expressible operations. Necessary and sufficient conditions are found for both kinds of expressibility. These conditions involve the enveloping algebra of the algebra formed by the original multiplication. Mutual expressibility, both simple and strong, is an equivalence relation and differs from isotopy. Reducibility is a property common to mutually expressible algebras. Associativity, commutativity, and the existence of nilpotents are properties which need not be preserved. (Received March 18, 1942.)

ANALYSIS

169. Dorothy L. Bernstein and S. M. Ulam: *On the problem of completely additive measure in classes of sets with a general equivalence relation.*

The problem of finding a necessary and sufficient condition for the existence of a finitely additive measure in a class of sets, with the property that equivalent sets have equal measure, has been solved by Tarski (*Fundamenta Mathematicae*, vol. 31 (1938), pp. 47-66). This paper considers the existence of a completely additive measure in a class K which is a Borel field over a given sequence $\{A_n\}$ of sets and which has the property that $m(X) = m(Y)$ if X is equivalent to Y . Equivalence is defined in the most general sense: Given a division of the sets of K into disjoint classes A_1, A_2, A_3, \dots , sets X and Y are called equivalent if they belong to the same class A_α . It should be noted that even if no requirement for equivalence is made, it is not always possible to find a measure in a Borel field over a given sequence of sets. A necessary and suffi-