ON INTEGRAL FUNCTIONS OF INTEGRAL OR ZERO ORDER

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Let $F(z)$ be an integral function of finite order ρ. We write $F(z)=z^k e^{g(z)}f(z)$ where $g(z)$ is a polynomial of degree $q\leqq\rho$ and

$$f(z) = \prod_1^\infty\left\{\left(1-\frac{z}{a_n}\right)\exp\left(\frac{z}{a_n}+\cdots+\frac{1}{p}\left(\frac{z}{a_n}\right)^p\right)\right\}$$

is the canonical product of order ρ_1 and genus p. Let $M(r, F) =\max_{|z|=r}|F(z)|$ and $n(r, F-a)=n(r, a)$ be the number of zeros of $F(z)-a$ in $|z|=r$. In an earlier paper[1] I proved the following result.

THEOREM 1. *If $F(z)$ be of integral order ρ and if the genus of the canonical product $f(z)$ be $p=\rho$, then*

$$\liminf_{r=\infty}\frac{\log M(r,F)}{n(r,F)\phi(r)}=0 \tag{1}$$

where $\phi(x)$ is any positive continuous increasing function of the real variable x such that

$$\int_a^\infty\frac{dx}{x\phi(x)} \tag{2}$$

is convergent.

In this note I prove a similar result for the canonical products of order ρ and genus $p=\rho-1$, and discuss whether the result can be extended to integral functions which are not canonical products. The main result is the following.

THEOREM 2. *If $f(z)$ is a canonical product of integral order ρ and genus $p=\rho-1$ then*

$$\liminf_{r=\infty}\frac{\log M(r,f)}{n(r,f)\,\Phi(r)}=0 \tag{3}$$

where $\Phi(x)$ is any positive increasing function such that

$$\int_a^\infty\frac{dx}{x\Phi(x)} \tag{4}$$

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[1] *A Theorem on integral functions of integral order*, Journal of the London Mathematical Society, vol. 15 (1940), pp. 23–31. I shall refer to this paper as (1).