quence of polynomials whose roots lie on the axis of pure imaginaries and which converges uniformly in every finite region.

HUNTER COLLEGE

GENERALIZED LAPLACE INTEGRALS

R. P. BOAS, JR.

We consider the linear space $\mathfrak{H}(c)$ whose elements are functions $f(z)$ $[z=x+iy]$ which are analytic for $x>c$ and satisfy

$$\int_{-\infty}^{\infty} |f(x+iy)|^2 dy \leqq M, \qquad x > c, \tag{1}$$

where the finite number M depends on the function in question. It is well known that an element $f(z)$ of $\mathfrak{H}(c)$ has boundary values $f(c+iy)$ almost everywhere on $x=c$, and that $\mathfrak{H}(c)$ is a Hilbert space if the norm of $f(z)$ is defined by

$$\|f(z)\|^2 = \int_{-\infty}^{\infty} |f(c+iy)|^2 dy.$$

Furthermore, it is known [5, p. 8] that if $f(z)\in\mathfrak{H}(c)$, then $f(z)$ is representable as a Laplace integral for $x>c$, in the sense that there is a unique function[1] $\phi(t)$ with $e^{-ct}\phi(t)\in L^2(0, \infty)$ such that

$$\lim_{T\to\infty} \left\| f(z) - \int_0^T e^{-zt}\phi(t)dt \right\| = 0; \tag{2}$$

we shall express (2) by writing

$$f(z) = \int_0^{\infty} e^{-zt}\phi(t)dt, \qquad x > c. \tag{3}$$

It is easily verified that the integral in (3) converges in the ordinary sense for $x>c$. A Laplace integral may be regarded as a generalized power series; the object of this note is to generalize the integral representation (3) by replacing e^{-zt} by a kernel $g(z, t)$ which is in some sense "nearly" e^{-zt}, just as power series $\sum a_n z^n$ have been generalized[2] by replacing the functions z^n by functions $g_n(z)$.

Presented to the Society, September 5, 1941; received by the editors May 24, 1941.

[1] Unique, that is, up to sets of measure zero.

[2] For a bibliography of this problem, see [1].